

Maple 2018.2 Integration Test Results  
on the problems in "8 Special functions"

Test results for the 82 problems in "8.1 Error functions.txt"

Problem 10: Unable to integrate problem.

$$\int x^3 \operatorname{erf}(bx)^2 dx$$

Optimal(type 4, 112 leaves, 8 steps):

$$\frac{1}{2b^4 e^{2b^2 x^2} \pi} + \frac{x^2}{4b^2 e^{2b^2 x^2} \pi} - \frac{3 \operatorname{erf}(bx)^2}{16b^4} + \frac{x^4 \operatorname{erf}(bx)^2}{4} + \frac{3x \operatorname{erf}(bx)}{4b^3 e^{b^2 x^2} \sqrt{\pi}} + \frac{x^3 \operatorname{erf}(bx)}{2b e^{b^2 x^2} \sqrt{\pi}}$$

Result(type 8, 12 leaves):

$$\int x^3 \operatorname{erf}(bx)^2 dx$$

Problem 12: Unable to integrate problem.

$$\int (dx+c)^2 \operatorname{erf}(bx+a)^2 dx$$

Optimal(type 4, 345 leaves, 16 steps):

$$\begin{aligned} & \frac{d(-ad+bc)}{b^3 e^{2(bx+a)^2} \pi} + \frac{d^2(bx+a)}{3b^3 e^{2(bx+a)^2} \pi} - \frac{d(-ad+bc) \operatorname{erf}(bx+a)^2}{2b^3} + \frac{(-ad+bc)^2 (bx+a) \operatorname{erf}(bx+a)^2}{b^3} + \frac{d(-ad+bc) (bx+a)^2 \operatorname{erf}(bx+a)^2}{b^3} \\ & + \frac{d^2(bx+a)^3 \operatorname{erf}(bx+a)^2}{3b^3} - \frac{(-ad+bc)^2 \operatorname{erf}((bx+a)\sqrt{2})\sqrt{2}}{\sqrt{\pi} b^3} + \frac{2d^2 \operatorname{erf}(bx+a)}{3b^3 e^{(bx+a)^2} \sqrt{\pi}} + \frac{2(-ad+bc)^2 \operatorname{erf}(bx+a)}{b^3 e^{(bx+a)^2} \sqrt{\pi}} \\ & + \frac{2d(-ad+bc) (bx+a) \operatorname{erf}(bx+a)}{b^3 e^{(bx+a)^2} \sqrt{\pi}} + \frac{2d^2 (bx+a)^2 \operatorname{erf}(bx+a)}{3b^3 e^{(bx+a)^2} \sqrt{\pi}} - \frac{5d^2 \operatorname{erf}((bx+a)\sqrt{2})\sqrt{2}}{12b^3 \sqrt{\pi}} \end{aligned}$$

Result(type 8, 18 leaves):

$$\int (dx+c)^2 \operatorname{erf}(bx+a)^2 dx$$

Problem 14: Unable to integrate problem.

$$\int x \operatorname{erf}(d(a+b \ln(cx^n))) dx$$

Optimal(type 4, 91 leaves, 5 steps):

$$\frac{x^2 \operatorname{erf}(d(a + b \ln(cx^n)))}{2} - \frac{e^{-\frac{2abd^2n+1}{b^2d^2n^2}} x^2 \operatorname{erf}\left(\frac{abd^2 - \frac{1}{n} + b^2d^2 \ln(cx^n)}{bd}\right)}{2(cx^n)^{\frac{2}{n}}}$$

Result(type 8, 17 leaves):

$$\int x \operatorname{erf}(d(a + b \ln(cx^n))) dx$$

Problem 15: Unable to integrate problem.

$$\int \frac{\operatorname{erf}(d(a + b \ln(cx^n)))}{x^3} dx$$

Optimal(type 4, 90 leaves, 5 steps):

$$-\frac{\operatorname{erf}(d(a + b \ln(cx^n)))}{2x^2} + \frac{e^{-\frac{2abd^2n+1}{b^2d^2n^2}} (cx^n)^{\frac{2}{n}} \operatorname{erf}\left(\frac{1 + abd^2n + b^2d^2n \ln(cx^n)}{bdn}\right)}{2x^2}$$

Result(type 8, 19 leaves):

$$\int \frac{\operatorname{erf}(d(a + b \ln(cx^n)))}{x^3} dx$$

Problem 17: Unable to integrate problem.

$$\int \frac{e^{-b^2x^2+c}}{\operatorname{erf}(bx)} dx$$

Optimal(type 4, 15 leaves, 2 steps):

$$\frac{e^c \ln(\operatorname{erf}(bx)) \sqrt{\pi}}{2b}$$

Result(type 8, 20 leaves):

$$\int \frac{e^{-b^2x^2+c}}{\operatorname{erf}(bx)} dx$$

Problem 23: Unable to integrate problem.

$$\int e^{b^2x^2+c} x^4 \operatorname{erf}(bx) dx$$

Optimal(type 5, 96 leaves, 7 steps):

$$-\frac{3 e^{b^2 x^2 + c} x \operatorname{erf}(b x)}{4 b^4} + \frac{e^{b^2 x^2 + c} x^3 \operatorname{erf}(b x)}{2 b^2} + \frac{3 e^c x^2}{4 b^3 \sqrt{\pi}} - \frac{e^c x^4}{4 b \sqrt{\pi}} + \frac{3 e^c x^2 \operatorname{HypergeometricPFQ}\left(\left[1, 1\right], \left[\frac{3}{2}, 2\right], b^2 x^2\right)}{4 b^3 \sqrt{\pi}}$$

Result(type 8, 20 leaves):

$$\int e^{b^2 x^2 + c} x^4 \operatorname{erf}(b x) dx$$

Problem 25: Unable to integrate problem.

$$\int \frac{x^4 \operatorname{erf}(b x)}{e^{b^2 x^2}} dx$$

Optimal(type 4, 98 leaves, 7 steps):

$$-\frac{3 x \operatorname{erf}(b x)}{4 b^4 e^{b^2 x^2}} - \frac{x^3 \operatorname{erf}(b x)}{2 b^2 e^{b^2 x^2}} - \frac{1}{2 b^5 e^2 b^2 x^2 \sqrt{\pi}} - \frac{x^2}{4 b^3 e^2 b^2 x^2 \sqrt{\pi}} + \frac{3 \operatorname{erf}(b x)^2 \sqrt{\pi}}{16 b^5}$$

Result(type 8, 20 leaves):

$$\int \frac{x^4 \operatorname{erf}(b x)}{e^{b^2 x^2}} dx$$

Problem 28: Unable to integrate problem.

$$\int \operatorname{erf}(b x) \sinh(b^2 x^2 + c) dx$$

Optimal(type 5, 44 leaves, 4 steps):

$$\frac{b e^c x^2 \operatorname{HypergeometricPFQ}\left(\left[1, 1\right], \left[\frac{3}{2}, 2\right], b^2 x^2\right)}{2 \sqrt{\pi}} - \frac{\operatorname{erf}(b x)^2 \sqrt{\pi}}{8 b e^c}$$

Result(type 8, 17 leaves):

$$\int \operatorname{erf}(b x) \sinh(b^2 x^2 + c) dx$$

Problem 29: Unable to integrate problem.

$$\int -\operatorname{erf}(b x) \sinh(b^2 x^2 - c) dx$$

Optimal(type 5, 44 leaves, 4 steps):

$$-\frac{b x^2 \operatorname{HypergeometricPFQ}\left(\left[1, 1\right], \left[\frac{3}{2}, 2\right], b^2 x^2\right)}{2 e^c \sqrt{\pi}} + \frac{e^c \operatorname{erf}(b x)^2 \sqrt{\pi}}{8 b}$$

Result(type 8, 20 leaves):

$$\int -\operatorname{erf}(bx) \sinh(b^2 x^2 - c) dx$$

Problem 30: Unable to integrate problem.

$$\int \cosh(b^2 x^2 + c) \operatorname{erf}(bx) dx$$

Optimal(type 5, 44 leaves, 4 steps):

$$\frac{b e^c x^2 \operatorname{HypergeometricPFQ}\left([1, 1], \left[\frac{3}{2}, 2\right], b^2 x^2\right)}{2 \sqrt{\pi}} + \frac{\operatorname{erf}(bx)^2 \sqrt{\pi}}{8 b e^c}$$

Result(type 8, 17 leaves):

$$\int \cosh(b^2 x^2 + c) \operatorname{erf}(bx) dx$$

Problem 33: Result more than twice size of optimal antiderivative.

$$\int (dx + c)^3 \operatorname{erfc}(bx + a) dx$$

Optimal(type 4, 260 leaves, 12 steps):

$$\begin{aligned} & \frac{3 d^3 \operatorname{erf}(bx + a)}{16 b^4} + \frac{3 d (-ad + bc)^2 \operatorname{erf}(bx + a)}{4 b^4} + \frac{(-ad + bc)^4 \operatorname{erf}(bx + a)}{4 b^4 d} + \frac{(dx + c)^4 \operatorname{erfc}(bx + a)}{4 d} - \frac{d^2 (-ad + bc)}{b^4 e^{(bx+a)^2} \sqrt{\pi}} - \frac{(-ad + bc)^3}{b^4 e^{(bx+a)^2} \sqrt{\pi}} \\ & - \frac{3 d^3 (bx + a)}{8 b^4 e^{(bx+a)^2} \sqrt{\pi}} - \frac{3 d (-ad + bc)^2 (bx + a)}{2 b^4 e^{(bx+a)^2} \sqrt{\pi}} - \frac{d^2 (-ad + bc) (bx + a)^2}{b^4 e^{(bx+a)^2} \sqrt{\pi}} - \frac{d^3 (bx + a)^3}{4 b^4 e^{(bx+a)^2} \sqrt{\pi}} \end{aligned}$$

Result(type 4, 728 leaves):

$$\begin{aligned} & \frac{1}{b} \left( \frac{d^3 \operatorname{erfc}(bx + a) (bx + a)^4}{4 b^3} - \frac{d^3 \operatorname{erfc}(bx + a) (bx + a)^3 a}{b^3} + \frac{d^2 \operatorname{erfc}(bx + a) (bx + a)^3 c}{b^2} + \frac{3 d^3 \operatorname{erfc}(bx + a) (bx + a)^2 a^2}{2 b^3} \right. \\ & - \frac{3 d^2 \operatorname{erfc}(bx + a) (bx + a)^2 a c}{b^2} + \frac{3 d \operatorname{erfc}(bx + a) (bx + a)^2 c^2}{2 b} - \frac{d^3 \operatorname{erfc}(bx + a) (bx + a) a^3}{b^3} + \frac{3 d^2 \operatorname{erfc}(bx + a) (bx + a) a^2 c}{b^2} \\ & - \frac{3 d \operatorname{erfc}(bx + a) (bx + a) a c^2}{b} + \operatorname{erfc}(bx + a) (bx + a) c^3 + \frac{d^3 \operatorname{erfc}(bx + a) a^4}{4 b^3} - \frac{d^2 \operatorname{erfc}(bx + a) a^3 c}{b^2} + \frac{3 d \operatorname{erfc}(bx + a) a^2 c^2}{2 b} - \operatorname{erfc}(bx \\ & + a) a c^3 + \frac{b \operatorname{erfc}(bx + a) c^4}{4 d} + \frac{1}{2 \sqrt{\pi} b^3 d} \left( d^4 \left( -\frac{(bx + a)^3}{2 e^{(bx+a)^2}} - \frac{3 (bx + a)}{4 e^{(bx+a)^2}} + \frac{3 \sqrt{\pi} \operatorname{erf}(bx + a)}{8} \right) + \frac{a^4 d^4 \sqrt{\pi} \operatorname{erf}(bx + a)}{2} \right) \end{aligned}$$



$$\begin{aligned}
& + \frac{b^4 c^4 \sqrt{\pi} \operatorname{erf}(bx+a)}{2} + \frac{2a^3 d^4}{e^{(bx+a)^2}} + 6a^2 d^4 \left( -\frac{bx+a}{2e^{(bx+a)^2}} + \frac{\sqrt{\pi} \operatorname{erf}(bx+a)}{4} \right) - 4a d^4 \left( -\frac{(bx+a)^2}{2e^{(bx+a)^2}} - \frac{1}{2e^{(bx+a)^2}} \right) - \frac{2b^3 c^3 d}{e^{(bx+a)^2}} + 6b^2 c^2 d^2 \left( -\frac{bx+a}{2e^{(bx+a)^2}} + \frac{\sqrt{\pi} \operatorname{erf}(bx+a)}{4} \right) + 4bc d^3 \left( -\frac{(bx+a)^2}{2e^{(bx+a)^2}} - \frac{1}{2e^{(bx+a)^2}} \right) - 2ab^3 c^3 d \sqrt{\pi} \operatorname{erf}(bx+a) + 3a^2 b^2 c^2 d^2 \sqrt{\pi} \operatorname{erf}(bx+a) \\
& - 2a^3 bc d^3 \sqrt{\pi} \operatorname{erf}(bx+a) + \frac{6ab^2 c^2 d^2}{e^{(bx+a)^2}} - \frac{6a^2 bc d^3}{e^{(bx+a)^2}} - 12abc d^3 \left( -\frac{bx+a}{2e^{(bx+a)^2}} + \frac{\sqrt{\pi} \operatorname{erf}(bx+a)}{4} \right) \Big) \Big)
\end{aligned}$$

Problem 34: Result more than twice size of optimal antiderivative.

$$\int (dx+c)^2 \operatorname{erfc}(bx+a) dx$$

Optimal (type 4, 172 leaves, 9 steps):

$$\begin{aligned}
& \frac{d(-ad+bc) \operatorname{erf}(bx+a)}{2b^3} + \frac{(-ad+bc)^3 \operatorname{erf}(bx+a)}{3b^3 d} + \frac{(dx+c)^3 \operatorname{erfc}(bx+a)}{3d} - \frac{d^2}{3b^3 e^{(bx+a)^2} \sqrt{\pi}} - \frac{(-ad+bc)^2}{b^3 e^{(bx+a)^2} \sqrt{\pi}} - \frac{d(-ad+bc)(bx+a)}{b^3 e^{(bx+a)^2} \sqrt{\pi}} \\
& - \frac{d^2 (bx+a)^2}{3b^3 e^{(bx+a)^2} \sqrt{\pi}}
\end{aligned}$$

Result (type 4, 427 leaves):

$$\begin{aligned}
& \frac{1}{b} \left( \frac{d^2 \operatorname{erfc}(bx+a) (bx+a)^3}{3b^2} - \frac{d^2 \operatorname{erfc}(bx+a) (bx+a)^2 a}{b^2} + \frac{d \operatorname{erfc}(bx+a) (bx+a)^2 c}{b} + \frac{d^2 \operatorname{erfc}(bx+a) (bx+a) a^2}{b^2} \right. \\
& - \frac{2d \operatorname{erfc}(bx+a) (bx+a) a c}{b} + \operatorname{erfc}(bx+a) (bx+a) c^2 - \frac{d^2 \operatorname{erfc}(bx+a) a^3}{3b^2} + \frac{d \operatorname{erfc}(bx+a) a^2 c}{b} - \operatorname{erfc}(bx+a) a c^2 \\
& + \frac{b \operatorname{erfc}(bx+a) c^3}{3d} + \frac{1}{3\sqrt{\pi} b^2 d} \left( 2 \left( d^3 \left( -\frac{(bx+a)^2}{2e^{(bx+a)^2}} - \frac{1}{2e^{(bx+a)^2}} \right) + \frac{b^3 c^3 \sqrt{\pi} \operatorname{erf}(bx+a)}{2} - \frac{a^3 d^3 \sqrt{\pi} \operatorname{erf}(bx+a)}{2} - \frac{3a^2 d^3}{2e^{(bx+a)^2}} - 3a d^3 \left( -\frac{bx+a}{2e^{(bx+a)^2}} + \frac{\sqrt{\pi} \operatorname{erf}(bx+a)}{4} \right) - \frac{3b^2 c^2 d}{2e^{(bx+a)^2}} + 3bc d^2 \left( -\frac{bx+a}{2e^{(bx+a)^2}} + \frac{\sqrt{\pi} \operatorname{erf}(bx+a)}{4} \right) - \frac{3ab^2 c^2 d \sqrt{\pi} \operatorname{erf}(bx+a)}{2} + \frac{3a^2 bc d^2 \sqrt{\pi} \operatorname{erf}(bx+a)}{2} + \frac{3abc d^2}{e^{(bx+a)^2}} \right) \right) \Big)
\end{aligned}$$

Problem 38: Unable to integrate problem.

$$\int x^3 \operatorname{erfc}(bx)^2 dx$$

Optimal (type 4, 112 leaves, 8 steps):

$$\frac{1}{2 b^4 e^{2 b^2 x^2} \pi} + \frac{x^2}{4 b^2 e^{2 b^2 x^2} \pi} - \frac{3 \operatorname{erfc}(b x)^2}{16 b^4} + \frac{x^4 \operatorname{erfc}(b x)^2}{4} - \frac{3 x \operatorname{erfc}(b x)}{4 b^3 e^{b^2 x^2} \sqrt{\pi}} - \frac{x^3 \operatorname{erfc}(b x)}{2 b e^{b^2 x^2} \sqrt{\pi}}$$

Result(type 8, 12 leaves):

$$\int x^3 \operatorname{erfc}(b x)^2 dx$$

Problem 39: Unable to integrate problem.

$$\int \frac{\operatorname{erfc}(b x)^2}{x^7} dx$$

Optimal(type 4, 157 leaves, 12 steps):

$$-\frac{b^2}{15 e^{2 b^2 x^2} \pi x^4} + \frac{2 b^4}{9 e^{2 b^2 x^2} \pi x^2} + \frac{28 b^6 \operatorname{Ei}(-2 b^2 x^2)}{45 \pi} - \frac{4 b^6 \operatorname{erfc}(b x)^2}{45} - \frac{\operatorname{erfc}(b x)^2}{6 x^6} + \frac{2 b \operatorname{erfc}(b x)}{15 e^{b^2 x^2} x^5 \sqrt{\pi}} - \frac{4 b^3 \operatorname{erfc}(b x)}{45 e^{b^2 x^2} x^3 \sqrt{\pi}} + \frac{8 b^5 \operatorname{erfc}(b x)}{45 e^{b^2 x^2} x \sqrt{\pi}}$$

Result(type 8, 12 leaves):

$$\int \frac{\operatorname{erfc}(b x)^2}{x^7} dx$$

Problem 42: Unable to integrate problem.

$$\int \frac{\operatorname{erfc}(d(a + b \ln(cx^n)))}{x^2} dx$$

Optimal(type 4, 88 leaves, 5 steps):

$$-\frac{\frac{1}{e^{4 b^2 d^2 n^2} + \frac{a}{b n}} (c x^n)^{\frac{1}{n}} \operatorname{erf}\left(\frac{2 a b d^2 + \frac{1}{n} + 2 b^2 d^2 \ln(cx^n)}{2 b d}\right)}{x} - \frac{\operatorname{erfc}(d(a + b \ln(cx^n)))}{x}$$

Result(type 8, 19 leaves):

$$\int \frac{\operatorname{erfc}(d(a + b \ln(cx^n)))}{x^2} dx$$

Problem 43: Unable to integrate problem.

$$\int \frac{\operatorname{erfc}(d(a + b \ln(cx^n)))}{x^3} dx$$

Optimal(type 4, 90 leaves, 5 steps):

$$-\frac{\frac{2 a b d^2 n + 1}{b^2 d^2 n^2} (c x^n)^{\frac{2}{n}} \operatorname{erf}\left(\frac{1 + a b d^2 n + b^2 d^2 n \ln(cx^n)}{b d n}\right)}{2 x^2} - \frac{\operatorname{erfc}(d(a + b \ln(cx^n)))}{2 x^2}$$

Result(type 8, 19 leaves):

$$\int \frac{\operatorname{erfc}(d(a + b \ln(cx^n)))}{x^3} dx$$

Problem 44: Unable to integrate problem.

$$\int \frac{e^{-b^2 x^2 + c}}{\operatorname{erfc}(bx)} dx$$

Optimal(type 4, 15 leaves, 2 steps):

$$-\frac{e^c \ln(\operatorname{erfc}(bx)) \sqrt{\pi}}{2b}$$

Result(type 8, 20 leaves):

$$\int \frac{e^{-b^2 x^2 + c}}{\operatorname{erfc}(bx)} dx$$

Problem 45: Unable to integrate problem.

$$\int e^{-b^2 x^2 + c} \operatorname{erfc}(bx)^n dx$$

Optimal(type 4, 23 leaves, 2 steps):

$$-\frac{e^c \operatorname{erfc}(bx)^{1+n} \sqrt{\pi}}{2b(1+n)}$$

Result(type 8, 20 leaves):

$$\int e^{-b^2 x^2 + c} \operatorname{erfc}(bx)^n dx$$

Problem 49: Unable to integrate problem.

$$\int \frac{\operatorname{erfc}(bx)}{e^{b^2 x^2} x^4} dx$$

Optimal(type 4, 93 leaves, 7 steps):

$$-\frac{\operatorname{erfc}(bx)}{3 e^{b^2 x^2} x^3} + \frac{2 b^2 \operatorname{erfc}(bx)}{3 e^{b^2 x^2} x} + \frac{b}{3 e^{2 b^2 x^2} x^2 \sqrt{\pi}} + \frac{4 b^3 \operatorname{Ei}(-2 b^2 x^2)}{3 \sqrt{\pi}} - \frac{b^3 \operatorname{erfc}(bx)^2 \sqrt{\pi}}{3}$$

Result(type 8, 20 leaves):

$$\int \frac{\operatorname{erfc}(bx)}{e^{b^2 x^2} x^4} dx$$

Problem 50: Result more than twice size of optimal antiderivative.

$$\int e^{dx^2+c} x \operatorname{erfc}(bx+a) dx$$

Optimal(type 4, 76 leaves, 3 steps):

$$\frac{e^{dx^2+c} \operatorname{erfc}(bx+a)}{2d} + \frac{b e^{c + \frac{a^2 d}{b^2 - d}} \operatorname{erf}\left(\frac{ab + (b^2 - d)x}{\sqrt{b^2 - d}}\right)}{2d\sqrt{b^2 - d}}$$

Result(type 4, 174 leaves):

$$\frac{1}{b} \left( \frac{b e^{\frac{(bx+a)^2 d}{b^2} - \frac{2ad(bx+a)}{b^2} + \frac{a^2 d}{b^2} + c}}{2d} - \frac{\operatorname{erf}(bx+a) b e^{\frac{(bx+a)^2 d}{b^2} - \frac{2ad(bx+a)}{b^2} + \frac{a^2 d}{b^2} + c}}{2d} \right. \\ \left. + \frac{b e^{\frac{a^2 d}{b^2} + c - \frac{a^2 d^2}{b^4 \left(-1 + \frac{d}{b^2}\right)}} \operatorname{erf}\left(\sqrt{1 - \frac{d}{b^2}} (bx+a) + \frac{ad}{b^2 \sqrt{1 - \frac{d}{b^2}}}\right)}{2d \sqrt{1 - \frac{d}{b^2}}} \right)$$

Problem 52: Unable to integrate problem.

$$\int -\operatorname{erfc}(bx) \sin(-c + I b^2 x^2) dx$$

Optimal(type 5, 70 leaves, 6 steps):

$$\frac{I b e^{I c} x^2 \operatorname{HypergeometricPFQ}\left([1, 1], \left[\frac{3}{2}, 2\right], b^2 x^2\right)}{2\sqrt{\pi}} - \frac{I \operatorname{erfc}(bx)^2 \sqrt{\pi}}{8 b e^{I c}} - \frac{I e^{I c} \operatorname{erfi}(bx) \sqrt{\pi}}{4 b}$$

Result(type 8, 22 leaves):

$$\int -\operatorname{erfc}(bx) \sin(-c + I b^2 x^2) dx$$

Problem 53: Unable to integrate problem.

$$\int \cosh(b^2 x^2 + c) \operatorname{erfc}(bx) dx$$

Optimal(type 5, 58 leaves, 6 steps):

$$-\frac{b e^c x^2 \text{HypergeometricPFQ}\left([1, 1], \left[\frac{3}{2}, 2\right], b^2 x^2\right)}{2\sqrt{\pi}} - \frac{\operatorname{erfc}(bx)^2 \sqrt{\pi}}{8 b e^c} + \frac{e^c \operatorname{erfi}(bx) \sqrt{\pi}}{4 b}$$

Result(type 8, 17 leaves):

$$\int \cosh(b^2 x^2 + c) \operatorname{erfc}(bx) dx$$

Problem 58: Result more than twice size of optimal antiderivative.

$$\int (dx + c)^3 \operatorname{erfi}(bx + a) dx$$

Optimal(type 4, 247 leaves, 12 steps):

$$\begin{aligned} & -\frac{3 d^3 \operatorname{erfi}(bx + a)}{16 b^4} + \frac{3 d (-ad + bc)^2 \operatorname{erfi}(bx + a)}{4 b^4} - \frac{(-ad + bc)^4 \operatorname{erfi}(bx + a)}{4 b^4 d} + \frac{(dx + c)^4 \operatorname{erfi}(bx + a)}{4 d} + \frac{d^2 (-ad + bc) e^{(bx+a)^2}}{b^4 \sqrt{\pi}} \\ & - \frac{(-ad + bc)^3 e^{(bx+a)^2}}{b^4 \sqrt{\pi}} + \frac{3 d^3 e^{(bx+a)^2} (bx + a)}{8 b^4 \sqrt{\pi}} - \frac{3 d (-ad + bc)^2 e^{(bx+a)^2} (bx + a)}{2 b^4 \sqrt{\pi}} - \frac{d^2 (-ad + bc) e^{(bx+a)^2} (bx + a)^2}{b^4 \sqrt{\pi}} \\ & - \frac{d^3 e^{(bx+a)^2} (bx + a)^3}{4 b^4 \sqrt{\pi}} \end{aligned}$$

Result(type 4, 702 leaves):

$$\begin{aligned} & \frac{1}{b} \left( \frac{d^3 \operatorname{erfi}(bx + a) (bx + a)^4}{4 b^3} - \frac{d^3 \operatorname{erfi}(bx + a) (bx + a)^3 a}{b^3} + \frac{d^2 \operatorname{erfi}(bx + a) (bx + a)^3 c}{b^2} + \frac{3 d^3 \operatorname{erfi}(bx + a) (bx + a)^2 a^2}{2 b^3} \right. \\ & - \frac{3 d^2 \operatorname{erfi}(bx + a) (bx + a)^2 a c}{b^2} + \frac{3 d \operatorname{erfi}(bx + a) (bx + a)^2 c^2}{2 b} - \frac{d^3 \operatorname{erfi}(bx + a) (bx + a) a^3}{b^3} + \frac{3 d^2 \operatorname{erfi}(bx + a) (bx + a) a^2 c}{b^2} \\ & - \frac{3 d \operatorname{erfi}(bx + a) (bx + a) a c^2}{b} + \operatorname{erfi}(bx + a) (bx + a) c^3 + \frac{d^3 \operatorname{erfi}(bx + a) a^4}{4 b^3} - \frac{d^2 \operatorname{erfi}(bx + a) a^3 c}{b^2} + \frac{3 d \operatorname{erfi}(bx + a) a^2 c^2}{2 b} - \operatorname{erfi}(bx + a) a c^3 \\ & + \frac{b \operatorname{erfi}(bx + a) c^4}{4 d} - \frac{1}{2 b^3 d \sqrt{\pi}} \left( d^4 \left( \frac{e^{(bx+a)^2} (bx + a)^3}{2} - \frac{3 (bx + a) e^{(bx+a)^2}}{4} + \frac{3 \sqrt{\pi} \operatorname{erfi}(bx + a)}{8} \right) + \frac{a^4 d^4 \sqrt{\pi} \operatorname{erfi}(bx + a)}{2} \right. \\ & \left. + \frac{b^4 c^4 \sqrt{\pi} \operatorname{erfi}(bx + a)}{2} - 2 a^3 d^4 e^{(bx+a)^2} + 6 a^2 d^4 \left( \frac{(bx + a) e^{(bx+a)^2}}{2} - \frac{\sqrt{\pi} \operatorname{erfi}(bx + a)}{4} \right) - 4 a d^4 \left( \frac{(bx + a)^2 e^{(bx+a)^2}}{2} - \frac{e^{(bx+a)^2}}{2} \right) \right) \end{aligned}$$

$$\begin{aligned}
& + 2b^3c^3de^{(bx+a)^2} + 6b^2c^2d^2 \left( \frac{(bx+a)e^{(bx+a)^2}}{2} - \frac{\sqrt{\pi} \operatorname{erfi}(bx+a)}{4} \right) + 4bcd^3 \left( \frac{(bx+a)^2e^{(bx+a)^2}}{2} - \frac{e^{(bx+a)^2}}{2} \right) - 2ab^3c^3d\sqrt{\pi} \operatorname{erfi}(bx+a) \\
& + 3a^2b^2c^2d^2\sqrt{\pi} \operatorname{erfi}(bx+a) - 2a^3bcd^3\sqrt{\pi} \operatorname{erfi}(bx+a) - 6ab^2c^2d^2e^{(bx+a)^2} + 6a^2bcd^3e^{(bx+a)^2} - 12abcd^3 \left( \frac{(bx+a)e^{(bx+a)^2}}{2} \right. \\
& \left. - \frac{\sqrt{\pi} \operatorname{erfi}(bx+a)}{4} \right) \Big) \Big) \Big)
\end{aligned}$$

Problem 60: Unable to integrate problem.

$$\int x^5 \operatorname{erfi}(bx)^2 dx$$

Optimal (type 4, 147 leaves, 12 steps):

$$\frac{11e^{2b^2x^2}}{12b^6\pi} - \frac{7e^{2b^2x^2}x^2}{12b^4\pi} + \frac{e^{2b^2x^2}x^4}{6b^2\pi} + \frac{5\operatorname{erfi}(bx)^2}{16b^6} + \frac{x^6\operatorname{erfi}(bx)^2}{6} - \frac{5e^{b^2x^2}x\operatorname{erfi}(bx)}{4b^5\sqrt{\pi}} + \frac{5e^{b^2x^2}x^3\operatorname{erfi}(bx)}{6b^3\sqrt{\pi}} - \frac{e^{b^2x^2}x^5\operatorname{erfi}(bx)}{3b\sqrt{\pi}}$$

Result (type 8, 12 leaves):

$$\int x^5 \operatorname{erfi}(bx)^2 dx$$

Problem 61: Unable to integrate problem.

$$\int x \operatorname{erfi}(bx)^2 dx$$

Optimal (type 4, 61 leaves, 5 steps):

$$\frac{e^{2b^2x^2}}{2b^2\pi} + \frac{\operatorname{erfi}(bx)^2}{4b^2} + \frac{x^2\operatorname{erfi}(bx)^2}{2} - \frac{e^{b^2x^2}x\operatorname{erfi}(bx)}{b\sqrt{\pi}}$$

Result (type 8, 10 leaves):

$$\int x \operatorname{erfi}(bx)^2 dx$$

Problem 63: Unable to integrate problem.

$$\int \frac{\operatorname{erfi}(bx)^2}{x^5} dx$$

Optimal (type 4, 104 leaves, 8 steps):

$$-\frac{b^2e^{2b^2x^2}}{3\pi x^2} + \frac{4b^4\operatorname{Ei}(2b^2x^2)}{3\pi} + \frac{b^4\operatorname{erfi}(bx)^2}{3} - \frac{\operatorname{erfi}(bx)^2}{4x^4} - \frac{be^{b^2x^2}\operatorname{erfi}(bx)}{3x^3\sqrt{\pi}} - \frac{2b^3e^{b^2x^2}\operatorname{erfi}(bx)}{3x\sqrt{\pi}}$$

Result (type 8, 12 leaves):

$$\int \frac{\operatorname{erfi}(bx)^2}{x^5} dx$$

Problem 66: Unable to integrate problem.

$$\int (dx + c) \operatorname{erfi}(bx + a)^2 dx$$

Optimal (type 4, 166 leaves, 10 steps):

$$\frac{d e^{2(bx+a)^2}}{2 b^2 \pi} + \frac{d \operatorname{erfi}(bx+a)^2}{4 b^2} + \frac{(-ad+bc)(bx+a) \operatorname{erfi}(bx+a)^2}{b^2} + \frac{d(bx+a)^2 \operatorname{erfi}(bx+a)^2}{2 b^2} + \frac{(-ad+bc) \operatorname{erfi}((bx+a)\sqrt{2})\sqrt{2}}{\sqrt{\pi} b^2}$$

$$- \frac{2(-ad+bc) e^{(bx+a)^2} \operatorname{erfi}(bx+a)}{b^2 \sqrt{\pi}} - \frac{d e^{(bx+a)^2} (bx+a) \operatorname{erfi}(bx+a)}{b^2 \sqrt{\pi}}$$

Result (type 8, 16 leaves):

$$\int (dx + c) \operatorname{erfi}(bx + a)^2 dx$$

Problem 67: Unable to integrate problem.

$$\int \operatorname{erfi}(bx + a)^2 dx$$

Optimal (type 4, 60 leaves, 4 steps):

$$\frac{(bx+a) \operatorname{erfi}(bx+a)^2}{b} + \frac{\operatorname{erfi}((bx+a)\sqrt{2})\sqrt{2}}{\sqrt{\pi} b} - \frac{2 e^{(bx+a)^2} \operatorname{erfi}(bx+a)}{b \sqrt{\pi}}$$

Result (type 8, 10 leaves):

$$\int \operatorname{erfi}(bx + a)^2 dx$$

Problem 68: Unable to integrate problem.

$$\int x \operatorname{erfi}(d(a + b \ln(cx^n))) dx$$

Optimal (type 4, 91 leaves, 5 steps):

$$\frac{x^2 \operatorname{erfi}(d(a + b \ln(cx^n)))}{2} - \frac{x^2 \operatorname{erfi}\left(\frac{ab d^2 + \frac{1}{n} + b^2 d^2 \ln(cx^n)}{bd}\right)}{2 e^{\frac{2abd^2n+1}{b^2 d^2 n^2}} (cx^n)^{\frac{2}{n}}}$$

Result (type 8, 17 leaves):

$$\int x \operatorname{erfi}(d(a + b \ln(cx^n))) dx$$

Problem 69: Unable to integrate problem.

$$\int \frac{\operatorname{erfi}(d(a + b \ln(cx^n)))}{x^2} dx$$

Optimal(type 4, 89 leaves, 5 steps):

$$-\frac{\operatorname{erfi}(d(a + b \ln(cx^n)))}{x} + \frac{e^{-\frac{1}{4b^2d^2n^2} + \frac{a}{bn}} (cx^n)^{\frac{1}{n}} \operatorname{erfi}\left(\frac{2abd^2 - \frac{1}{n} + 2b^2d^2 \ln(cx^n)}{2bd}\right)}{x}$$

Result(type 8, 19 leaves):

$$\int \frac{\operatorname{erfi}(d(a + b \ln(cx^n)))}{x^2} dx$$

Problem 70: Unable to integrate problem.

$$\int e^{b^2x^2+c} \operatorname{erfi}(bx)^n dx$$

Optimal(type 4, 23 leaves, 2 steps):

$$\frac{e^c \operatorname{erfi}(bx)^{1+n} \sqrt{\pi}}{2b(1+n)}$$

Result(type 8, 19 leaves):

$$\int e^{b^2x^2+c} \operatorname{erfi}(bx)^n dx$$

Problem 71: Unable to integrate problem.

$$\int e^{dx^2+c} x^5 \operatorname{erfi}(bx) dx$$

Optimal(type 4, 220 leaves, 9 steps):

$$\frac{e^{dx^2+c} \operatorname{erfi}(bx)}{d^3} - \frac{e^{dx^2+c} x^2 \operatorname{erfi}(bx)}{d^2} + \frac{e^{dx^2+c} x^4 \operatorname{erfi}(bx)}{2d} - \frac{3be^c \operatorname{erfi}(x\sqrt{b^2+d})}{8d(b^2+d)^{5/2}} - \frac{be^c \operatorname{erfi}(x\sqrt{b^2+d})}{2d^2(b^2+d)^{3/2}} - \frac{be^c \operatorname{erfi}(x\sqrt{b^2+d})}{d^3\sqrt{b^2+d}}$$

$$+ \frac{3be^{c+(b^2+d)x^2} x}{4d(b^2+d)^2\sqrt{\pi}} + \frac{be^{c+(b^2+d)x^2} x}{d^2(b^2+d)\sqrt{\pi}} - \frac{be^{c+(b^2+d)x^2} x^3}{2d(b^2+d)\sqrt{\pi}}$$

Result(type 8, 18 leaves):

$$\int e^{dx^2+c} x^5 \operatorname{erfi}(bx) dx$$

Problem 74: Unable to integrate problem.



$$\int \frac{x^2 \operatorname{erfi}(bx)}{e^{b^2 x^2}} dx$$

Optimal(type 5, 58 leaves, 3 steps):

$$-\frac{x \operatorname{erfi}(bx)}{2 b^2 e^{b^2 x^2}} + \frac{x^2}{2 b \sqrt{\pi}} + \frac{x^2 \operatorname{HypergeometricPFQ}\left(\left[1, 1\right], \left[\frac{3}{2}, 2\right], -b^2 x^2\right)}{2 b \sqrt{\pi}}$$

Result(type 8, 20 leaves):

$$\int \frac{x^2 \operatorname{erfi}(bx)}{e^{b^2 x^2}} dx$$

Problem 75: Unable to integrate problem.

$$\int \frac{\operatorname{erfi}(bx)}{e^{b^2 x^2} x^4} dx$$

Optimal(type 5, 87 leaves, 5 steps):

$$-\frac{\operatorname{erfi}(bx)}{3 e^{b^2 x^2} x^3} + \frac{2 b^2 \operatorname{erfi}(bx)}{3 e^{b^2 x^2} x} - \frac{b}{3 x^2 \sqrt{\pi}} + \frac{4 b^5 x^2 \operatorname{HypergeometricPFQ}\left(\left[1, 1\right], \left[\frac{3}{2}, 2\right], -b^2 x^2\right)}{3 \sqrt{\pi}} - \frac{4 b^3 \ln(x)}{3 \sqrt{\pi}}$$

Result(type 8, 20 leaves):

$$\int \frac{\operatorname{erfi}(bx)}{e^{b^2 x^2} x^4} dx$$

Problem 76: Unable to integrate problem.

$$\int \frac{\operatorname{erfi}(bx)}{e^{b^2 x^2} x^6} dx$$

Optimal(type 5, 120 leaves, 7 steps):

$$-\frac{\operatorname{erfi}(bx)}{5 e^{b^2 x^2} x^5} + \frac{2 b^2 \operatorname{erfi}(bx)}{15 e^{b^2 x^2} x^3} - \frac{4 b^4 \operatorname{erfi}(bx)}{15 e^{b^2 x^2} x} - \frac{b}{10 x^4 \sqrt{\pi}} + \frac{2 b^3}{15 x^2 \sqrt{\pi}} - \frac{8 b^7 x^2 \operatorname{HypergeometricPFQ}\left(\left[1, 1\right], \left[\frac{3}{2}, 2\right], -b^2 x^2\right)}{15 \sqrt{\pi}} + \frac{8 b^5 \ln(x)}{15 \sqrt{\pi}}$$

Result(type 8, 20 leaves):

$$\int \frac{\operatorname{erfi}(bx)}{e^{b^2 x^2} x^6} dx$$

Problem 77: Unable to integrate problem.

$$\int e^{b^2 x^2 + c} x^3 \operatorname{erfi}(b x) \, dx$$

Optimal(type 4, 79 leaves, 5 steps):

$$-\frac{e^{b^2 x^2 + c} \operatorname{erfi}(b x)}{2 b^4} + \frac{e^{b^2 x^2 + c} x^2 \operatorname{erfi}(b x)}{2 b^2} + \frac{5 e^c \operatorname{erfi}(b x \sqrt{2}) \sqrt{2}}{16 b^4} - \frac{e^{2 b^2 x^2 + c} x}{4 b^3 \sqrt{\pi}}$$

Result(type 8, 20 leaves):

$$\int e^{b^2 x^2 + c} x^3 \operatorname{erfi}(b x) \, dx$$

Problem 78: Unable to integrate problem.

$$\int e^{b^2 x^2 + c} x^4 \operatorname{erfi}(b x) \, dx$$

Optimal(type 4, 100 leaves, 7 steps):

$$-\frac{3 e^{b^2 x^2 + c} x \operatorname{erfi}(b x)}{4 b^4} + \frac{e^{b^2 x^2 + c} x^3 \operatorname{erfi}(b x)}{2 b^2} + \frac{e^{2 b^2 x^2 + c}}{2 b^5 \sqrt{\pi}} - \frac{e^{2 b^2 x^2 + c} x^2}{4 b^3 \sqrt{\pi}} + \frac{3 e^c \operatorname{erfi}(b x)^2 \sqrt{\pi}}{16 b^5}$$

Result(type 8, 20 leaves):

$$\int e^{b^2 x^2 + c} x^4 \operatorname{erfi}(b x) \, dx$$

Problem 79: Unable to integrate problem.

$$\int e^{d x^2 + c} x \operatorname{erfi}(b x + a) \, dx$$

Optimal(type 4, 68 leaves, 3 steps):

$$\frac{e^{d x^2 + c} \operatorname{erfi}(b x + a)}{2 d} - \frac{b e^{c + \frac{a^2 d}{b^2 + d}} \operatorname{erfi}\left(\frac{a b + (b^2 + d) x}{\sqrt{b^2 + d}}\right)}{2 d \sqrt{b^2 + d}}$$

Result(type 8, 18 leaves):

$$\int e^{d x^2 + c} x \operatorname{erfi}(b x + a) \, dx$$

Problem 82: Unable to integrate problem.

$$\int -\operatorname{erfi}(b x) \sinh(b^2 x^2 - c) \, dx$$

Optimal(type 5, 45 leaves, 4 steps):

$$\frac{b e^c x^2 \text{HypergeometricPFQ}\left(\left[1, 1\right], \left[\frac{3}{2}, 2\right], -b^2 x^2\right)}{2 \sqrt{\pi}} - \frac{\operatorname{erfi}(b x)^2 \sqrt{\pi}}{8 b e^c}$$

Result(type 8, 20 leaves):

$$\int -\operatorname{erfi}(b x) \sinh(b^2 x^2 - c) dx$$

Test results for the 27 problems in "8.10 Formal derivatives.txt"

Problem 11: Result more than twice size of optimal antiderivative.

$$\int \frac{-g(x) \operatorname{Derivative}(1)(f)(x) - f(x) \operatorname{Derivative}(1)(g)(x)}{1 + f(x)^2 g(x)^2} dx$$

Optimal(type 9, 8 leaves, 2 steps):

$$-\arctan(f(x) g(x))$$

Result(type 9, 29 leaves):

$$\int \frac{-g(x) \operatorname{Derivative}(1)(f)(x) - f(x) \operatorname{Derivative}(1)(g)(x)}{1 + f(x)^2 g(x)^2} dx$$

Problem 23: Result more than twice size of optimal antiderivative.

$$\int \cos(\operatorname{Derivative}(-1+m)(f)(x) \operatorname{Derivative}(-1+n)(g)(x)) (\operatorname{Derivative}(m)(f)(x) \operatorname{Derivative}(-1+n)(g)(x) + \operatorname{Derivative}(-1+m)(f)(x) \operatorname{Derivative}(n)(g)(x)) dx$$

Optimal(type 9, 6 leaves, 2 steps):

$$\sin(\operatorname{Derivative}(-1+m)(f)(x) \operatorname{Derivative}(-1+n)(g)(x))$$

Result(type 9, 20 leaves):

$$\int \cos(\operatorname{Derivative}(-1+m)(f)(x) \operatorname{Derivative}(-1+n)(g)(x)) (\operatorname{Derivative}(m)(f)(x) \operatorname{Derivative}(-1+n)(g)(x) + \operatorname{Derivative}(-1+m)(f)(x) \operatorname{Derivative}(n)(g)(x)) dx$$

Problem 24: Result more than twice size of optimal antiderivative.

$$\int \cos(\operatorname{Derivative}(-1+m)(f)(x)^2 \operatorname{Derivative}(-1+n)(g)(x)) \operatorname{Derivative}(-1+m)(f)(x) (2 \operatorname{Derivative}(m)(f)(x) \operatorname{Derivative}(-1+n)(g)(x) + \operatorname{Derivative}(-1+m)(f)(x) \operatorname{Derivative}(n)(g)(x)) dx$$

Optimal(type 9, 8 leaves, 2 steps):

$$\sin(\operatorname{Derivative}(-1+m)(f)(x)^2 \operatorname{Derivative}(-1+n)(g)(x))$$

Result(type 9, 25 leaves):

$$\int \cos(\operatorname{Derivative}(-1+m)(f)(x)^2 \operatorname{Derivative}(-1+n)(g)(x)) \operatorname{Derivative}(-1+m)(f)(x) (2 \operatorname{Derivative}(m)(f)(x) \operatorname{Derivative}(-1+n)(g)(x) + \operatorname{Derivative}(-1+m)(f)(x) \operatorname{Derivative}(n)(g)(x)) dx$$

$-1 + m) (f) (x) \text{Derivative}(n) (g) (x) \text{ dx}$

Problem 26: Result more than twice size of optimal antiderivative.

$$\int \cos(\text{Derivative}(m) (f) (x)^2 \text{Derivative}(n) (g) (x)^3) \text{Derivative}(m) (f) (x) \text{Derivative}(n) (g) (x)^2 (2 \text{Derivative}(1 + m) (f) (x) \text{Derivative}(n) (g) (x) + 3 \text{Derivative}(m) (f) (x) \text{Derivative}(1 + n) (g) (x)) \text{ dx}$$

Optimal(type 9, 10 leaves, 2 steps):

$$\sin(\text{Derivative}(m) (f) (x)^2 \text{Derivative}(n) (g) (x)^3)$$

Result(type 9, 32 leaves):

$$\int \cos(\text{Derivative}(m) (f) (x)^2 \text{Derivative}(n) (g) (x)^3) \text{Derivative}(m) (f) (x) \text{Derivative}(n) (g) (x)^2 (2 \text{Derivative}(1 + m) (f) (x) \text{Derivative}(n) (g) (x) + 3 \text{Derivative}(m) (f) (x) \text{Derivative}(1 + n) (g) (x)) \text{ dx}$$

Test results for the 56 problems in "8.2 Fresnel integral functions.txt"

Problem 12: Unable to integrate problem.

$$\int x^3 \text{FresnelS}(bx)^2 \text{ dx}$$

Optimal(type 4, 122 leaves, 10 steps):

$$\frac{3x^2}{8b^2\pi^2} + \frac{x^2 \cos(b^2\pi x^2)}{8b^2\pi^2} + \frac{x^3 \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelS}(bx)}{2b\pi} + \frac{3 \text{FresnelS}(bx)^2}{4b^4\pi^2} + \frac{x^4 \text{FresnelS}(bx)^2}{4} - \frac{3x \text{FresnelS}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{2b^3\pi^2} - \frac{\sin(b^2\pi x^2)}{2b^4\pi^3}$$

Result(type 8, 12 leaves):

$$\int x^3 \text{FresnelS}(bx)^2 \text{ dx}$$

Problem 16: Unable to integrate problem.

$$\int \frac{\text{FresnelS}(bx)^2}{x^9} \text{ dx}$$

Optimal(type 4, 210 leaves, 20 steps):

$$-\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} + \frac{b^2 \cos(b^2\pi x^2)}{336x^6} - \frac{b^6\pi^2 \cos(b^2\pi x^2)}{336x^2} - \frac{b^3\pi \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelS}(bx)}{140x^5} + \frac{b^7\pi^3 \cos\left(\frac{b^2\pi x^2}{2}\right) \text{FresnelS}(bx)}{420x} + \frac{b^8\pi^4 \text{FresnelS}(bx)^2}{840} - \frac{\text{FresnelS}(bx)^2}{8x^8} - \frac{b^8\pi^3 \text{Si}(b^2\pi x^2)}{280} - \frac{b \text{FresnelS}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{28x^7} + \frac{b^5\pi^2 \text{FresnelS}(bx) \sin\left(\frac{b^2\pi x^2}{2}\right)}{420x^3} - \frac{b^4\pi \sin(b^2\pi x^2)}{420x^4}$$

Result(type 8, 12 leaves):

$$\int \frac{\text{FresnelS}(bx)^2}{x^9} dx$$

Problem 17: Unable to integrate problem.

$$\int (dx+c)^2 \text{FresnelS}(bx+a)^2 dx$$

Optimal (type 5, 451 leaves, 18 steps):

$$\begin{aligned} & \frac{2d^2x}{3b^2\pi^2} + \frac{d(-ad+bc)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{2b^3\pi^2} + \frac{d^2(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{6b^3\pi^2} + \frac{2(-ad+bc)^2\cos\left(\frac{\pi(bx+a)^2}{2}\right)\text{FresnelS}(bx+a)}{b^3\pi} \\ & + \frac{2d(-ad+bc)(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)\text{FresnelS}(bx+a)}{b^3\pi} + \frac{2d^2(bx+a)^2\cos\left(\frac{\pi(bx+a)^2}{2}\right)\text{FresnelS}(bx+a)}{3b^3\pi} \\ & - \frac{d(-ad+bc)\text{FresnelC}(bx+a)\text{FresnelS}(bx+a)}{b^3\pi} + \frac{(-ad+bc)^2(bx+a)\text{FresnelS}(bx+a)^2}{b^3} + \frac{d(-ad+bc)(bx+a)^2\text{FresnelS}(bx+a)^2}{b^3} \\ & + \frac{d^2(bx+a)^3\text{FresnelS}(bx+a)^2}{3b^3} + \frac{\text{Id}(-ad+bc)(bx+a)^2\text{HypergeometricPFQ}\left([1,1],\left[\frac{3}{2},2\right],-\frac{1}{2}\pi(bx+a)^2\right)}{4b^3\pi} \\ & - \frac{\text{Id}(-ad+bc)(bx+a)^2\text{HypergeometricPFQ}\left([1,1],\left[\frac{3}{2},2\right],\frac{1}{2}\pi(bx+a)^2\right)}{4b^3\pi} - \frac{4d^2\text{FresnelS}(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{3b^3\pi^2} \\ & - \frac{5d^2\text{FresnelC}\left((bx+a)\sqrt{2}\right)\sqrt{2}}{12b^3\pi^2} - \frac{(-ad+bc)^2\text{FresnelS}\left((bx+a)\sqrt{2}\right)\sqrt{2}}{2b^3\pi} \end{aligned}$$

Result (type 8, 18 leaves):

$$\int (dx+c)^2 \text{FresnelS}(bx+a)^2 dx$$

Problem 18: Unable to integrate problem.

$$\int \text{FresnelS}(d(a+b\ln(cx^n))) dx$$

Optimal (type 4, 186 leaves, 10 steps):

$$\frac{\left(\frac{1}{4}-\frac{1}{4}\right)x\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{1}{2}\right)\left(\frac{1}{n}+Iab d^2\pi+Ib^2 d^2\pi\ln(cx^n)\right)}{bd\sqrt{\pi}}\right)}{e^{\frac{2abn-\frac{1}{\pi d^2}}{2b^2n^2}}(cx^n)^{\frac{1}{n}}} + \frac{\left(\frac{1}{4}-\frac{1}{4}\right)x\text{erfi}\left(\frac{\left(\frac{1}{2}+\frac{1}{2}\right)\left(\frac{1}{n}-Iab d^2\pi-Ib^2 d^2\pi\ln(cx^n)\right)}{bd\sqrt{\pi}}\right)}{e^{\frac{2abn+\frac{1}{\pi d^2}}{2b^2n^2}}(cx^n)^{\frac{1}{n}}}$$

+ x FresnelS(d (a + b ln(cx^n)))

Result(type 8, 15 leaves):

$$\int \text{FresnelS}(d (a + b \ln(cx^n))) dx$$

Problem 19: Unable to integrate problem.

$$\int \frac{\text{FresnelS}(d (a + b \ln(cx^n)))}{x^3} dx$$

Optimal(type 4, 200 leaves, 10 steps):

$$\frac{\left(\frac{1}{8} - \frac{I}{8}\right) e^{\frac{2I+2ab d^2 n \pi}{b^2 d^2 n^2 \pi}} (cx^n)^{\frac{2}{n}} \operatorname{erf}\left(\frac{\left(\frac{1}{2} + \frac{I}{2}\right) \left(\frac{2}{n} - Iab d^2 \pi - Ib^2 d^2 \pi \ln(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x^2} + \frac{\left(\frac{1}{8} - \frac{I}{8}\right) (cx^n)^{\frac{2}{n}} \operatorname{erfi}\left(\frac{\left(\frac{1}{2} + \frac{I}{2}\right) \left(\frac{2}{n} + Iab d^2 \pi + Ib^2 d^2 \pi \ln(cx^n)\right)}{bd\sqrt{\pi}}\right)}{e^{\frac{2(1-ab d^2 n \pi)}{b^2 d^2 n^2 \pi}} x^2} - \frac{\text{FresnelS}(d (a + b \ln(cx^n)))}{2x^2}$$

Result(type 8, 19 leaves):

$$\int \frac{\text{FresnelS}(d (a + b \ln(cx^n)))}{x^3} dx$$

Problem 20: Unable to integrate problem.

$$\int \cos\left(c + \frac{b^2 \pi x^2}{2}\right) \text{FresnelS}(bx) dx$$

Optimal(type 5, 81 leaves, 4 steps):

$$\frac{\cos(c) \text{FresnelC}(bx) \text{FresnelS}(bx)}{2b} - \frac{Ibx^2 \cos(c) \text{HypergeometricPFQ}\left(\left[1, 1\right], \left[\frac{3}{2}, 2\right], -\frac{1}{2} b^2 \pi x^2\right)}{8} + \frac{Ibx^2 \cos(c) \text{HypergeometricPFQ}\left(\left[1, 1\right], \left[\frac{3}{2}, 2\right], \frac{1}{2} b^2 \pi x^2\right)}{8} - \frac{\text{FresnelS}(bx)^2 \sin(c)}{2b}$$

Result(type 8, 19 leaves):

$$\int \cos\left(c + \frac{b^2 \pi x^2}{2}\right) \text{FresnelS}(bx) dx$$

Problem 26: Unable to integrate problem.

$$\int x^8 \cos\left(\frac{b^2 \pi x^2}{2}\right) \text{FresnelS}(bx) \, dx$$

Optimal (type 5, 271 leaves, 23 steps):

$$\begin{aligned} & \frac{35x^4}{8b^5\pi^3} - \frac{x^8}{16b\pi} - \frac{40\cos(b^2\pi x^2)}{b^9\pi^5} + \frac{5x^4\cos(b^2\pi x^2)}{2b^5\pi^3} - \frac{105x\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelS}(bx)}{b^8\pi^4} + \frac{7x^5\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelS}(bx)}{b^4\pi^2} \\ & + \frac{105\text{FresnelC}(bx)\text{FresnelS}(bx)}{2b^9\pi^4} - \frac{105\text{I}x^2\text{HypergeometricPFQ}\left(\left[1, 1\right], \left[\frac{3}{2}, 2\right], -\frac{1}{2}b^2\pi x^2\right)}{8b^7\pi^4} \\ & + \frac{105\text{I}x^2\text{HypergeometricPFQ}\left(\left[1, 1\right], \left[\frac{3}{2}, 2\right], \frac{1}{2}b^2\pi x^2\right)}{8b^7\pi^4} - \frac{35x^3\text{FresnelS}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{b^6\pi^3} + \frac{x^7\text{FresnelS}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{b^2\pi} - \frac{55x^2\sin(b^2\pi x^2)}{4b^7\pi^4} \\ & + \frac{x^6\sin(b^2\pi x^2)}{4b^3\pi^2} \end{aligned}$$

Result (type 8, 20 leaves):

$$\int x^8 \cos\left(\frac{b^2 \pi x^2}{2}\right) \text{FresnelS}(bx) \, dx$$

Problem 29: Unable to integrate problem.

$$\int \frac{\cos\left(\frac{b^2 \pi x^2}{2}\right) \text{FresnelS}(bx)}{x^2} \, dx$$

Optimal (type 4, 42 leaves, 4 steps):

$$-\frac{\cos\left(\frac{b^2 \pi x^2}{2}\right) \text{FresnelS}(bx)}{x} - \frac{b\pi \text{FresnelS}(bx)^2}{2} + \frac{b \text{Si}(b^2 \pi x^2)}{4}$$

Result (type 8, 20 leaves):

$$\int \frac{\cos\left(\frac{b^2 \pi x^2}{2}\right) \text{FresnelS}(bx)}{x^2} \, dx$$

Problem 41: Unable to integrate problem.

$$\int x^5 \text{FresnelC}(bx)^2 \, dx$$

Optimal(type 5, 227 leaves, 16 steps):

$$\begin{aligned} & \frac{5x^4}{24b^2\pi^2} + \frac{11\cos(b^2\pi x^2)}{6b^6\pi^4} - \frac{x^4\cos(b^2\pi x^2)}{12b^2\pi^2} - \frac{5x^3\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelC}(bx)}{3b^3\pi^2} + \frac{x^6\text{FresnelC}(bx)^2}{6} - \frac{5\text{FresnelC}(bx)\text{FresnelS}(bx)}{2b^6\pi^3} \\ & - \frac{5I x^2 \text{HypergeometricPFQ}\left([1, 1], \left[\frac{3}{2}, 2\right], -\frac{1}{2}b^2\pi x^2\right)}{8b^4\pi^3} + \frac{5I x^2 \text{HypergeometricPFQ}\left([1, 1], \left[\frac{3}{2}, 2\right], \frac{1}{2}b^2\pi x^2\right)}{8b^4\pi^3} + \frac{5x\text{FresnelC}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{b^5\pi^3} \\ & - \frac{x^5\text{FresnelC}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{3b\pi} + \frac{7x^2\sin(b^2\pi x^2)}{12b^4\pi^3} \end{aligned}$$

Result(type 8, 12 leaves):

$$\int x^5 \text{FresnelC}(bx)^2 dx$$

Problem 44: Unable to integrate problem.

$$\int \frac{\text{FresnelC}(bx)^2}{x^5} dx$$

Optimal(type 4, 109 leaves, 9 steps):

$$\begin{aligned} & -\frac{b^2}{24x^2} - \frac{b^2\cos(b^2\pi x^2)}{24x^2} - \frac{b\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelC}(bx)}{6x^3} - \frac{b^4\pi^2\text{FresnelC}(bx)^2}{12} - \frac{\text{FresnelC}(bx)^2}{4x^4} - \frac{b^4\pi\text{Si}(b^2\pi x^2)}{12} \\ & + \frac{b^3\pi\text{FresnelC}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{6x} \end{aligned}$$

Result(type 8, 12 leaves):

$$\int \frac{\text{FresnelC}(bx)^2}{x^5} dx$$

Problem 46: Unable to integrate problem.

$$\int \frac{\text{FresnelC}(bx)^2}{x^9} dx$$

Optimal(type 4, 210 leaves, 20 steps):

$$\begin{aligned} & -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} - \frac{b^2\cos(b^2\pi x^2)}{336x^6} + \frac{b^6\pi^2\cos(b^2\pi x^2)}{336x^2} - \frac{b\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelC}(bx)}{28x^7} + \frac{b^5\pi^2\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelC}(bx)}{420x^3} + \frac{b^8\pi^4\text{FresnelC}(bx)^2}{840} \end{aligned}$$



$$-\frac{\text{FresnelC}(bx)^2}{8x^8} + \frac{b^8 \pi^3 \text{Si}(b^2 \pi x^2)}{280} + \frac{b^3 \pi \text{FresnelC}(bx) \sin\left(\frac{b^2 \pi x^2}{2}\right)}{140x^5} - \frac{b^7 \pi^3 \text{FresnelC}(bx) \sin\left(\frac{b^2 \pi x^2}{2}\right)}{420x} + \frac{b^4 \pi \sin(b^2 \pi x^2)}{420x^4}$$

Result(type 8, 12 leaves):

$$\int \frac{\text{FresnelC}(bx)^2}{x^9} dx$$

Problem 47: Unable to integrate problem.

$$\int \text{FresnelC}(d(a + b \ln(cx^n))) dx$$

Optimal(type 4, 186 leaves, 10 steps):

$$\frac{\left(\frac{1}{4} + \frac{1}{4}\right) x \operatorname{erf}\left(\frac{\left(\frac{1}{2} + \frac{1}{2}\right) \left(\frac{1}{n} + I a b d^2 \pi + I b^2 d^2 \pi \ln(cx^n)\right)}{b d \sqrt{\pi}}\right)}{e^{\frac{2 a b n - \frac{1}{\pi d^2}}{2 b^2 n^2}} (c x^n)^{\frac{1}{n}}}} - \frac{\left(\frac{1}{4} + \frac{1}{4}\right) x \operatorname{erfi}\left(\frac{\left(\frac{1}{2} + \frac{1}{2}\right) \left(\frac{1}{n} - I a b d^2 \pi - I b^2 d^2 \pi \ln(cx^n)\right)}{b d \sqrt{\pi}}\right)}{e^{\frac{2 a b n + \frac{1}{\pi d^2}}{2 b^2 n^2}} (c x^n)^{\frac{1}{n}}}} + x \text{FresnelC}(d(a + b \ln(cx^n)))$$

Result(type 8, 15 leaves):

$$\int \text{FresnelC}(d(a + b \ln(cx^n))) dx$$

Problem 48: Unable to integrate problem.

$$\int \frac{\text{FresnelC}(d(a + b \ln(cx^n)))}{x^2} dx$$

Optimal(type 4, 185 leaves, 10 steps):

$$\frac{\left(\frac{1}{4} + \frac{1}{4}\right) e^{\frac{2 a b n + \frac{1}{\pi d^2}}{2 b^2 n^2}} (c x^n)^{\frac{1}{n}} \operatorname{erf}\left(\frac{\left(\frac{1}{2} + \frac{1}{2}\right) \left(\frac{1}{n} - I a b d^2 \pi - I b^2 d^2 \pi \ln(cx^n)\right)}{b d \sqrt{\pi}}\right)}{x} - \frac{\left(\frac{1}{4} + \frac{1}{4}\right) e^{\frac{2 a b n - \frac{1}{\pi d^2}}{2 b^2 n^2}} (c x^n)^{\frac{1}{n}} \operatorname{erfi}\left(\frac{\left(\frac{1}{2} + \frac{1}{2}\right) \left(\frac{1}{n} + I a b d^2 \pi + I b^2 d^2 \pi \ln(cx^n)\right)}{b d \sqrt{\pi}}\right)}{x} - \frac{\text{FresnelC}(d(a + b \ln(cx^n)))}{x}$$

Result(type 8, 19 leaves):

$$\int \frac{\text{FresnelC}(d(a + b \ln(cx^n)))}{x^2} dx$$

Problem 52: Unable to integrate problem.

$$\int x^8 \text{FresnelC}(bx) \sin\left(\frac{b^2 \pi x^2}{2}\right) dx$$

Optimal(type 5, 272 leaves, 23 steps):

$$\begin{aligned} & -\frac{35x^4}{8b^5\pi^3} + \frac{x^8}{16b\pi} - \frac{40\cos(b^2\pi x^2)}{b^9\pi^5} + \frac{5x^4\cos(b^2\pi x^2)}{2b^5\pi^3} + \frac{35x^3\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelC}(bx)}{b^6\pi^3} - \frac{x^7\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelC}(bx)}{b^2\pi} \\ & + \frac{105\text{FresnelC}(bx)\text{FresnelS}(bx)}{2b^9\pi^4} + \frac{105Ix^2\text{HypergeometricPFQ}\left([1, 1], \left[\frac{3}{2}, 2\right], -\frac{1}{2}b^2\pi x^2\right)}{8b^7\pi^4} \\ & - \frac{105Ix^2\text{HypergeometricPFQ}\left([1, 1], \left[\frac{3}{2}, 2\right], \frac{1}{2}b^2\pi x^2\right)}{8b^7\pi^4} - \frac{105x\text{FresnelC}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{b^8\pi^4} + \frac{7x^5\text{FresnelC}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{b^4\pi^2} \\ & - \frac{55x^2\sin(b^2\pi x^2)}{4b^7\pi^4} + \frac{x^6\sin(b^2\pi x^2)}{4b^3\pi^2} \end{aligned}$$

Result(type 8, 20 leaves):

$$\int x^8 \text{FresnelC}(bx) \sin\left(\frac{b^2 \pi x^2}{2}\right) dx$$

Problem 55: Unable to integrate problem.

$$\int \text{FresnelC}(bx) \sin\left(\frac{b^2 \pi x^2}{2}\right) dx$$

Optimal(type 5, 62 leaves, 1 step):

$$\frac{\text{FresnelC}(bx)\text{FresnelS}(bx)}{2b} + \frac{1bx^2\text{HypergeometricPFQ}\left([1, 1], \left[\frac{3}{2}, 2\right], -\frac{1}{2}b^2\pi x^2\right)}{8} - \frac{1bx^2\text{HypergeometricPFQ}\left([1, 1], \left[\frac{3}{2}, 2\right], \frac{1}{2}b^2\pi x^2\right)}{8}$$

Result(type 8, 17 leaves):

$$\int \text{FresnelC}(bx) \sin\left(\frac{b^2 \pi x^2}{2}\right) dx$$

Test results for the 55 problems in "8.3 Exponential integral functions.txt"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int x^2 \text{Ei}_1(bx) \, dx$$

Optimal(type 4, 21 leaves, 1 step):

$$-\frac{x^3 \text{Ei}_{-2}(bx)}{3} + \frac{x^3 \text{Ei}_1(bx)}{3}$$

Result(type 4, 47 leaves):

$$\frac{\frac{b^3 x^3 \text{Ei}_1(bx)}{3} - \frac{b^2 x^2 e^{-bx}}{3} - \frac{2bx e^{-bx}}{3} - \frac{2e^{-bx}}{3}}{b^3}$$

Problem 3: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Ei}_1(bx)}{x^4} \, dx$$

Optimal(type 4, 21 leaves, 1 step):

$$-\frac{\text{Ei}_1(bx)}{3x^3} + \frac{\text{Ei}_4(bx)}{3x^3}$$

Result(type 4, 64 leaves):

$$b^3 \left( -\frac{\text{Ei}_1(bx)}{3b^3 x^3} + \frac{e^{-bx}}{9b^3 x^3} - \frac{e^{-bx}}{18b^2 x^2} + \frac{e^{-bx}}{18bx} - \frac{\text{Ei}_1(bx)}{18} \right)$$

Problem 4: Result more than twice size of optimal antiderivative.

$$\int \text{Ei}_2(bx) \, dx$$

Optimal(type 4, 10 leaves, 1 step):

$$-\frac{\text{Ei}_3(bx)}{b}$$

Result(type 4, 67 leaves):

$$\frac{\left( \gamma - \frac{3}{2} + \ln(x) + \ln(b) \right) x^2 b^2}{2} + \frac{3b^2 x^2}{4} + \frac{1}{2} - \frac{(-3bx + 3)e^{-bx}}{6} + \frac{b^2 x^2 (-\gamma - \ln(bx) - \text{Ei}_1(bx))}{2}$$


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$$b$$

Problem 5: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Ei}_2(bx)}{x^5} \, dx$$

Optimal(type 4, 21 leaves, 1 step):

$$-\frac{\text{Ei}_2(bx)}{3x^4} + \frac{\text{Ei}_5(bx)}{3x^4}$$

Result(type 4, 164 leaves):

$$b^4 \left( -\frac{1}{4b^4x^4} - \frac{-\frac{2}{3} + \gamma + \ln(x) + \ln(b)}{3x^3b^3} + \frac{1}{4b^2x^2} - \frac{1}{12bx} + \frac{29}{864} - \frac{\gamma}{72} - \frac{\ln(x)}{72} - \frac{\ln(b)}{72} + \frac{-145b^4x^4 + 360b^3x^3 - 1080b^2x^2 - 960bx + 1080}{4320b^4x^4} \right. \\ \left. - \frac{(20b^3x^3 - 20b^2x^2 + 40bx + 360)e^{-bx}}{1440b^4x^4} - \frac{(20b^3x^3 + 480)(-\gamma - \ln(bx) - \text{Ei}_1(bx))}{1440b^3x^3} \right)$$

Problem 6: Result more than twice size of optimal antiderivative.

$$\int x^2 \text{Ei}_3(bx) dx$$

Optimal(type 4, 21 leaves, 1 step):

$$-\frac{x^3 \text{Ei}_{-2}(bx)}{5} + \frac{x^3 \text{Ei}_3(bx)}{5}$$

Result(type 4, 91 leaves):

$$\frac{\left( -\frac{17}{10} + \gamma + \ln(x) + \ln(b) \right) x^5 b^5}{10} - \frac{17b^5x^5}{100} + \frac{2}{5} - \frac{(18b^4x^4 - 18b^3x^3 + 36b^2x^2 + 72bx + 72)e^{-bx}}{180} - \frac{b^5x^5(-\gamma - \ln(bx) - \text{Ei}_1(bx))}{10}$$


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$$b^3$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int x \text{Ei}_3(bx) dx$$

Optimal(type 4, 21 leaves, 1 step):

$$-\frac{x^2 \text{Ei}_{-1}(bx)}{4} + \frac{x^2 \text{Ei}_3(bx)}{4}$$

Result(type 4, 83 leaves):

$$\frac{\left( -\frac{7}{4} + \gamma + \ln(x) + \ln(b) \right) x^4 b^4}{8} - \frac{7b^4x^4}{32} + \frac{1}{4} - \frac{(15b^3x^3 - 15b^2x^2 + 30bx + 30)e^{-bx}}{120} - \frac{b^4x^4(-\gamma - \ln(bx) - \text{Ei}_1(bx))}{8}$$


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$$b^2$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int \text{Ei}_3(bx) dx$$

Optimal(type 4, 10 leaves, 1 step):

$$-\frac{\text{Ei}_4(bx)}{b}$$

Result(type 4, 75 leaves):

$$-\frac{\left(\gamma - \frac{11}{6} + \ln(x) + \ln(b)\right)x^3 b^3}{6} - \frac{11b^3 x^3}{36} + \frac{1}{3} - \frac{(4b^2 x^2 - 4bx + 8)e^{-bx}}{24} - \frac{b^3 x^3 (-\gamma - \ln(bx) - \text{Ei}_1(bx))}{6}$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Ei}_3(bx)}{x} dx$$

Optimal(type 4, 15 leaves, 1 step):

$$-\frac{\text{Ei}_1(bx)}{2} + \frac{\text{Ei}_3(bx)}{2}$$

Result(type 4, 77 leaves):

$$\frac{\gamma}{2} + \frac{\ln(x)}{2} + \frac{\ln(b)}{2} - \frac{(-2 + \gamma + \ln(x) + \ln(b))x^2 b^2}{4} - \frac{b^2 x^2}{2} + \frac{(-9bx + 9)e^{-bx}}{36} + \frac{(-9b^2 x^2 + 18)(-\gamma - \ln(bx) - \text{Ei}_1(bx))}{36}$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Ei}_3(bx)}{x^5} dx$$

Optimal(type 4, 21 leaves, 1 step):

$$-\frac{\text{Ei}_3(bx)}{2x^4} + \frac{\text{Ei}_5(bx)}{2x^4}$$

Result(type 4, 164 leaves):

$$b^4 \left( -\frac{1}{8b^4 x^4} + \frac{1}{3x^3 b^3} + \frac{-1 + \gamma + \ln(x) + \ln(b)}{4x^2 b^2} - \frac{1}{6bx} + \frac{31}{576} - \frac{\gamma}{48} - \frac{\ln(x)}{48} - \frac{\ln(b)}{48} + \frac{-155b^4 x^4 + 480b^3 x^3 + 720b^2 x^2 - 960bx + 360}{2880b^4 x^4} \right. \\ \left. - \frac{(15b^3 x^3 - 15b^2 x^2 - 150bx + 90)e^{-bx}}{720b^4 x^4} + \frac{(-15b^2 x^2 + 180)(-\gamma - \ln(bx) - \text{Ei}_1(bx))}{720b^2 x^2} \right)$$

Problem 11: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Ei}_{-1}(bx)}{x} dx$$

Optimal(type 4, 15 leaves, 1 step):

$$-\frac{\text{Ei}_{-1}(bx)}{2} + \frac{\text{Ei}_1(bx)}{2}$$

Result(type 4, 68 leaves):

$$-\frac{1}{2b^2x^2} + \frac{1}{4} - \frac{\ln(x)}{2} - \frac{\ln(b)}{2} + \frac{-3b^2x^2 + 6}{12b^2x^2} - \frac{(3bx + 3)e^{-bx}}{6b^2x^2} + \frac{\ln(bx)}{2} + \frac{\text{Ei}_1(bx)}{2}$$

Problem 13: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Ei}_{-2}(bx)}{x^3} dx$$

Optimal(type 4, 21 leaves, 1 step):

$$-\frac{\text{Ei}_{-2}(bx)}{5x^2} + \frac{\text{Ei}_3(bx)}{5x^2}$$

Result(type 4, 128 leaves):

$$b^2 \left( -\frac{2}{5x^5b^5} + \frac{1}{6b^2x^2} - \frac{1}{4bx} + \frac{17}{100} - \frac{\ln(x)}{10} - \frac{\ln(b)}{10} + \frac{-153b^5x^5 + 225b^4x^4 - 150b^3x^3 + 360}{900b^5x^5} - \frac{(18b^4x^4 - 18b^3x^3 + 36b^2x^2 + 72bx + 72)e^{-bx}}{180b^5x^5} + \frac{\ln(bx)}{10} + \frac{\text{Ei}_1(bx)}{10} \right)$$

Problem 15: Result more than twice size of optimal antiderivative.

$$\int \text{Ei}_{-1}(bx) dx$$

Optimal(type 3, 14 leaves, 1 step):

$$-\frac{1}{b^2e^{bx}x}$$

Result(type 3, 41 leaves):

$$\frac{-\frac{1}{bx} + 1 + \frac{-2bx + 2}{2bx} - \frac{e^{-bx}}{bx}}{b}$$

Problem 16: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Ei}_{-3}(bx)}{x^2} dx$$

Optimal(type 4, 21 leaves, 1 step):

$$-\frac{\text{Ei}_{-3}(bx)}{5x} + \frac{\text{Ei}_2(bx)}{5x}$$

Result(type 4, 110 leaves):

$$b \left( -\frac{6}{5x^5b^5} + \frac{1}{4bx} - \frac{6}{25} + \frac{\ln(x)}{5} + \frac{\ln(b)}{5} + \frac{72b^5x^5 - 75b^4x^4 + 360}{300b^5x^5} - \frac{(-12b^4x^4 + 12b^3x^3 + 36b^2x^2 + 72bx + 72)e^{-bx}}{60b^5x^5} - \frac{\ln(bx)}{5} - \frac{\text{Ei}_1(bx)}{5} \right)$$

Problem 20: Result unnecessarily involves higher level functions.

$$\int (dx)^m \text{Ei}_n(bx) dx$$

Optimal(type 4, 46 leaves, 1 step):

$$-\frac{(dx)^{1+m} \text{Ei}_{-m}(bx)}{d(m+n)} + \frac{(dx)^{1+m} \text{Ei}_n(bx)}{d(m+n)}$$

Result(type 5, 88 leaves):

$$(dx)^m x^{-m} b^{-1-m} \left( \frac{x^{1+m} b^{1+m} \text{hypergeom}([1+m, 1-n], [2+m, 2-n], -bx)}{(-1+n)(1+m)} + \frac{\pi x^{m+n} b^{m+n} \csc(\pi n)}{(m+n)\Gamma(n)} \right)$$

Problem 23: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x \text{Ei}_n(bx) dx$$

Optimal(type 4, 30 leaves, 1 step):

$$-\frac{x^2 \text{Ei}_{-1}(bx)}{1+n} + \frac{x^2 \text{Ei}_n(bx)}{1+n}$$

Result(type 5, 62 leaves):

$$\frac{\frac{b^2 x^2 \text{hypergeom}([2, 1-n], [3, 2-n], -bx)}{2(-1+n)} + \frac{\pi x^{1+n} b^{1+n} \csc(\pi n)}{(1+n)\Gamma(n)}}{b^2}$$

Problem 26: Unable to integrate problem.

$$\int (dx+c)^3 \text{Ei}_3(bx+a) dx$$

Optimal(type 4, 75 leaves, 4 steps):

$$-\frac{(dx+c)^3 \text{Ei}_4(bx+a)}{b} - \frac{3d(dx+c)^2 \text{Ei}_5(bx+a)}{b^2} - \frac{6d^2(dx+c) \text{Ei}_6(bx+a)}{b^3} - \frac{6d^3 \text{Ei}_7(bx+a)}{b^4}$$

Result(type 8, 17 leaves):

$$\int (dx+c)^3 \text{Ei}_3(bx+a) dx$$

Problem 27: Unable to integrate problem.

$$\int (dx + c)^2 \text{Ei}_3(bx + a) dx$$

Optimal(type 4, 53 leaves, 3 steps):

$$-\frac{(dx + c)^2 \text{Ei}_4(bx + a)}{b} - \frac{2d(dx + c) \text{Ei}_5(bx + a)}{b^2} - \frac{2d^2 \text{Ei}_6(bx + a)}{b^3}$$

Result(type 8, 17 leaves):

$$\int (dx + c)^2 \text{Ei}_3(bx + a) dx$$

Problem 28: Unable to integrate problem.

$$\int (dx + c)^3 \text{Ei}_{-1}(bx + a) dx$$

Optimal(type 4, 118 leaves, 7 steps):

$$-\frac{3d^3 e^{-bx-a}}{b^4} - \frac{3d^2(-ad + bc) e^{-bx-a}}{b^4} - \frac{3d^2 e^{-bx-a}(dx + c)}{b^3} - \frac{e^{-bx-a}(dx + c)^3}{b(bx + a)} + \frac{3d(-ad + bc)^2 \text{Ei}(-bx - a)}{b^4}$$

Result(type 8, 17 leaves):

$$\int (dx + c)^3 \text{Ei}_{-1}(bx + a) dx$$

Problem 29: Unable to integrate problem.

$$\int (dx + c)^2 \text{Ei}_{-1}(bx + a) dx$$

Optimal(type 4, 69 leaves, 5 steps):

$$-\frac{2d^2 e^{-bx-a}}{b^3} - \frac{e^{-bx-a}(dx + c)^2}{b(bx + a)} + \frac{2d(-ad + bc) \text{Ei}(-bx - a)}{b^3}$$

Result(type 8, 17 leaves):

$$\int (dx + c)^2 \text{Ei}_{-1}(bx + a) dx$$

Problem 30: Unable to integrate problem.

$$\int (dx + c) \text{Ei}_{-1}(bx + a) dx$$

Optimal(type 4, 41 leaves, 2 steps):

$$-\frac{e^{-bx-a}(dx + c)}{b(bx + a)} + \frac{d \text{Ei}(-bx - a)}{b^2}$$

Result(type 8, 15 leaves):

$$\int (dx + c) \text{Ei}_{-1}(bx + a) dx$$



Problem 31: Unable to integrate problem.

$$\int \frac{\text{Ei}_{-2}(bx+a)}{(dx+c)^2} dx$$

Optimal(type 4, 411 leaves, 15 steps):

$$\begin{aligned} & \frac{2d^2 e^{-bx-a}}{b^2(-ad+bc)(dx+c)^3} + \frac{2d e^{-bx-a}}{b^2(bx+a)(dx+c)^3} + \frac{3d^2 e^{-bx-a}}{b(-ad+bc)^2(dx+c)^2} - \frac{d e^{-bx-a}}{b(-ad+bc)(dx+c)^2} + \frac{6d^2 e^{-bx-a}}{(-ad+bc)^3(dx+c)} \\ & - \frac{3d e^{-bx-a}}{(-ad+bc)^2(dx+c)} + \frac{e^{-bx-a}}{(-ad+bc)(dx+c)} + \frac{6bd^2 \text{Ei}(-bx-a)}{(-ad+bc)^4} - \frac{6bd^2 e^{-a+\frac{bc}{d}} \text{Ei}\left(-\frac{b(dx+c)}{d}\right)}{(-ad+bc)^4} + \frac{6bde^{-a+\frac{bc}{d}} \text{Ei}\left(-\frac{b(dx+c)}{d}\right)}{(-ad+bc)^3} \\ & - \frac{3be^{-a+\frac{bc}{d}} \text{Ei}\left(-\frac{b(dx+c)}{d}\right)}{(-ad+bc)^2} + \frac{be^{-a+\frac{bc}{d}} \text{Ei}\left(-\frac{b(dx+c)}{d}\right)}{d(-ad+bc)} - \frac{\text{Ei}_{-1}(bx+a)}{b(dx+c)^2} \end{aligned}$$

Result(type 8, 17 leaves):

$$\int \frac{\text{Ei}_{-2}(bx+a)}{(dx+c)^2} dx$$

Problem 32: Unable to integrate problem.

$$\int (dx+c)^2 \text{Ei}_{-3}(bx+a) dx$$

Optimal(type 4, 62 leaves, 3 steps):

$$-\frac{2d^2 e^{-bx-a}}{b^3(bx+a)} - \frac{(dx+c)^2 \text{Ei}_{-2}(bx+a)}{b} - \frac{2d(dx+c) \text{Ei}_{-1}(bx+a)}{b^2}$$

Result(type 8, 17 leaves):

$$\int (dx+c)^2 \text{Ei}_{-3}(bx+a) dx$$

Problem 33: Unable to integrate problem.

$$\int (dx+c) \text{Ei}_{-3}(bx+a) dx$$

Optimal(type 4, 31 leaves, 2 steps):

$$-\frac{(dx+c) \text{Ei}_{-2}(bx+a)}{b} - \frac{d \text{Ei}_{-1}(bx+a)}{b^2}$$

Result(type 8, 15 leaves):

$$\int (dx+c) \text{Ei}_{-3}(bx+a) dx$$

Problem 34: Unable to integrate problem.

$$\int \text{Ei}_{-3}(bx+a) dx$$

Optimal(type 4, 12 leaves, 1 step):

$$-\frac{\text{Ei}_{-2}(bx+a)}{b}$$

Result(type 8, 9 leaves):

$$\int \text{Ei}_{-3}(bx+a) dx$$

Problem 35: Unable to integrate problem.

$$\int \frac{\text{Ei}_{-3}(bx+a)}{dx+c} dx$$

Optimal(type 4, 442 leaves, 16 steps):

$$\begin{aligned} & -\frac{2d^3 e^{-bx-a}}{b^3(-ad+bc)(dx+c)^3} - \frac{2d^2 e^{-bx-a}}{b^3(bx+a)(dx+c)^3} - \frac{3d^3 e^{-bx-a}}{b^2(-ad+bc)^2(dx+c)^2} + \frac{d^2 e^{-bx-a}}{b^2(-ad+bc)(dx+c)^2} - \frac{6d^3 e^{-bx-a}}{b(-ad+bc)^3(dx+c)} \\ & + \frac{3d^2 e^{-bx-a}}{b(-ad+bc)^2(dx+c)} - \frac{d e^{-bx-a}}{b(-ad+bc)(dx+c)} - \frac{6d^3 \text{Ei}(-bx-a)}{(-ad+bc)^4} + \frac{6d^3 e^{-a+\frac{bc}{d}} \text{Ei}\left(-\frac{b(dx+c)}{d}\right)}{(-ad+bc)^4} - \frac{6d^2 e^{-a+\frac{bc}{d}} \text{Ei}\left(-\frac{b(dx+c)}{d}\right)}{(-ad+bc)^3} \\ & + \frac{3d e^{-a+\frac{bc}{d}} \text{Ei}\left(-\frac{b(dx+c)}{d}\right)}{(-ad+bc)^2} - \frac{e^{-a+\frac{bc}{d}} \text{Ei}\left(-\frac{b(dx+c)}{d}\right)}{-ad+bc} - \frac{\text{Ei}_{-2}(bx+a)}{b(dx+c)} + \frac{d \text{Ei}_{-1}(bx+a)}{b^2(dx+c)^2} \end{aligned}$$

Result(type 8, 17 leaves):

$$\int \frac{\text{Ei}_{-3}(bx+a)}{dx+c} dx$$

Problem 39: Unable to integrate problem.

$$\int (dx+c) \text{Ei}_n(bx+a) dx$$

Optimal(type 4, 35 leaves, 2 steps):

$$-\frac{(dx+c) \text{Ei}_{1+n}(bx+a)}{b} - \frac{d \text{Ei}_{2+n}(bx+a)}{b^2}$$

Result(type 8, 15 leaves):

$$\int (dx+c) \text{Ei}_n(bx+a) dx$$

Problem 50: Result more than twice size of optimal antiderivative.

$$\int e^{bx+a} x^2 \text{Ei}(dx+c) dx$$

Optimal(type 4, 228 leaves, 14 steps):

$$\begin{aligned} & \frac{e^{a+c+(b+d)x}}{b(b+d)^2} + \frac{2e^{a+c+(b+d)x}}{b^2(b+d)} + \frac{ce^{a+c+(b+d)x}}{bd(b+d)} - \frac{e^{a+c+(b+d)x}x}{b(b+d)} + \frac{2e^{bx+a}\text{Ei}(dx+c)}{b^3} - \frac{2e^{bx+a}x\text{Ei}(dx+c)}{b^2} + \frac{e^{bx+a}x^2\text{Ei}(dx+c)}{b} \\ & - \frac{2e^{a-\frac{bc}{d}}\text{Ei}\left(\frac{(b+d)(dx+c)}{d}\right)}{b^3} - \frac{c^2e^{a-\frac{bc}{d}}\text{Ei}\left(\frac{(b+d)(dx+c)}{d}\right)}{bd^2} - \frac{2ce^{a-\frac{bc}{d}}\text{Ei}\left(\frac{(b+d)(dx+c)}{d}\right)}{b^2d} \end{aligned}$$

Result(type 4, 693 leaves):

$$\begin{aligned} & \frac{1}{d} \left( \frac{1}{db} \left( \text{Ei}(dx \right. \right. \\ & \left. \left. + c) \left( \frac{d^2 \left( \left( \frac{b(dx+c)}{d} + \frac{ad-bc}{d} \right)^2 e^{\frac{b(dx+c)}{d} + \frac{ad-bc}{d}} - 2 \left( \frac{b(dx+c)}{d} + \frac{ad-bc}{d} \right) e^{\frac{b(dx+c)}{d} + \frac{ad-bc}{d}} + 2e^{\frac{b(dx+c)}{d} + \frac{ad-bc}{d}} \right)}{b^2} \right. \right. \\ & \left. \left. + \frac{e^{\frac{b(dx+c)}{d} + \frac{ad-bc}{d}} d^2 a^2 - 2d^2 a \left( \left( \frac{b(dx+c)}{d} + \frac{ad-bc}{d} \right) e^{\frac{b(dx+c)}{d} + \frac{ad-bc}{d}} - e^{\frac{b(dx+c)}{d} + \frac{ad-bc}{d}} \right)}{b^2} \right) \right) \\ & - \frac{1}{db} \left( \frac{1}{(b+d)^2} \left( d^2 \left( \left( \frac{(b+d)(dx+c)}{d} + \frac{ad-bc}{d} \right) e^{\frac{(b+d)(dx+c)}{d} + \frac{ad-bc}{d}} - e^{\frac{(b+d)(dx+c)}{d} + \frac{ad-bc}{d}} + \frac{(b+d)(dx+c)}{d} + \frac{ad-bc}{d} \right) \right. \right. \\ & \left. \left. - e^{\frac{(b+d)(dx+c)}{d} + \frac{ad-bc}{d}} a \right) \right) - c^2 e^{-\frac{ad+bc}{d}} \text{Ei}_1 \left( -\frac{(b+d)(dx+c)}{d} - \frac{ad-bc}{d} - \frac{-ad+bc}{d} \right) - \frac{2ce^{dx+c+\frac{b(dx+c)}{d}+a-\frac{bc}{d}}}{1+\frac{b}{d}} \\ & - \frac{2de^{dx+c+\frac{b(dx+c)}{d}+a-\frac{bc}{d}}}{b \left( 1 + \frac{b}{d} \right)} - \frac{2d^2 e^{-\frac{-ad+bc}{d}} \text{Ei}_1 \left( -\frac{(b+d)(dx+c)}{d} - \frac{ad-bc}{d} - \frac{-ad+bc}{d} \right)}{b^2} \end{aligned}$$

$$- \frac{2 d c e^{-\frac{-a d+b c}{d}} \operatorname{Ei}_1\left(-\frac{(b+d)(d x+c)}{d}-\frac{a d-b c}{d}-\frac{-a d+b c}{d}\right)}{b}$$

Test results for the 40 problems in "8.4 Trig integral functions.txt"

Problem 1: Result unnecessarily involves higher level functions.

$$\int x^m \operatorname{Si}(b x) dx$$

Optimal(type 4, 78 leaves, 5 steps):

$$\frac{x^m \Gamma(1+m, -I b x)}{2 b (1+m) (-I b x)^m} + \frac{x^m \Gamma(1+m, I b x)}{2 b (1+m) (I b x)^m} + \frac{x^{1+m} \operatorname{Si}(b x)}{1+m}$$

Result(type 5, 36 leaves):

$$\frac{b x^{2+m} \operatorname{hypergeom}\left(\left[\frac{1}{2}, 1+\frac{m}{2}\right], \left[\frac{3}{2}, \frac{3}{2}, 2+\frac{m}{2}\right], -\frac{b^2 x^2}{4}\right)}{2+m}$$

Problem 13: Unable to integrate problem.

$$\int \frac{\operatorname{Si}(b x) \sin(b x)}{x^3} dx$$

Optimal(type 4, 84 leaves, 14 steps):

$$b^2 \operatorname{Ci}(2 b x) - \frac{b \cos(b x) \operatorname{Si}(b x)}{2 x} - \frac{b^2 \operatorname{Si}(b x)^2}{4} - \frac{b \cos(b x) \sin(b x)}{2 x} - \frac{\operatorname{Si}(b x) \sin(b x)}{2 x^2} - \frac{\sin(b x)^2}{4 x^2} - \frac{b \sin(2 b x)}{4 x}$$

Result(type 8, 14 leaves):

$$\int \frac{\operatorname{Si}(b x) \sin(b x)}{x^3} dx$$

Problem 16: Unable to integrate problem.

$$\int \frac{\cos(b x) \operatorname{Si}(b x)}{x^2} dx$$

Optimal(type 4, 40 leaves, 7 steps):

$$b \operatorname{Ci}(2 b x) - \frac{\cos(b x) \operatorname{Si}(b x)}{x} - \frac{b \operatorname{Si}(b x)^2}{2} - \frac{\sin(2 b x)}{2 x}$$

Result(type 8, 14 leaves):

$$\int \frac{\cos(bx) \operatorname{Si}(bx)}{x^2} dx$$

Problem 22: Result more than twice size of optimal antiderivative.

$$\int x \cos(bx + a) \operatorname{Si}(dx + c) dx$$

Optimal (type 4, 350 leaves, 24 steps):

$$\begin{aligned} & \frac{c \operatorname{Ci}\left(\frac{c(b-d)}{d} + (b-d)x\right) \cos\left(a - \frac{bc}{d}\right)}{2bd} - \frac{c \operatorname{Ci}\left(\frac{c(b+d)}{d} + (b+d)x\right) \cos\left(a - \frac{bc}{d}\right)}{2bd} + \frac{\cos\left(a - \frac{bc}{d}\right) \operatorname{Si}\left(\frac{c(b-d)}{d} + (b-d)x\right)}{2b^2} \\ & + \frac{\cos(bx+a) \operatorname{Si}(dx+c)}{b^2} - \frac{\cos\left(a - \frac{bc}{d}\right) \operatorname{Si}\left(\frac{c(b+d)}{d} + (b+d)x\right)}{2b^2} + \frac{\operatorname{Ci}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b^2} \\ & - \frac{\operatorname{Ci}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b^2} - \frac{c \operatorname{Si}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2bd} + \frac{c \operatorname{Si}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2bd} \\ & + \frac{x \operatorname{Si}(dx+c) \sin(bx+a)}{b} - \frac{\sin(a-c+(b-d)x)}{2b(b-d)} + \frac{\sin(a+c+(b+d)x)}{2b(b+d)} \end{aligned}$$

Result (type 4, 1207 leaves):

$$\begin{aligned} & \frac{1}{d} \left( \frac{1}{b} \left( \operatorname{Si}(dx+c) \left( \frac{d \left( \cos\left(\frac{b(dx+c)}{d} + \frac{ad-bc}{d}\right) + \left(\frac{b(dx+c)}{d} + \frac{ad-bc}{d}\right) \sin\left(\frac{b(dx+c)}{d} + \frac{ad-bc}{d}\right) \right)}{b} \right. \right. \right. \\ & \left. \left. - \frac{da \sin\left(\frac{b(dx+c)}{d} + \frac{ad-bc}{d}\right)}{b} \right) \right) - \frac{1}{b} \left( \frac{d \sin\left(\frac{(b-d)(dx+c)}{d} + \frac{ad-bc}{d}\right)}{2(b-d)} + \frac{1}{2(b-d)} \left( ad \right. \right. \\ & \left. \left. - bc \right) d \left( \frac{\operatorname{Si}\left(\frac{(b-d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \sin\left(\frac{-ad+bc}{d}\right)}{d} \right. \right. \\ & \left. \left. + \frac{\operatorname{Ci}\left(\frac{(b-d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \cos\left(\frac{-ad+bc}{d}\right)}{d} \right) \right) \\ & - \frac{1}{2(b-d)} \left( ad^2 \left( \frac{\operatorname{Si}\left(\frac{(b-d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \sin\left(\frac{-ad+bc}{d}\right)}{d} \right. \right. \end{aligned}$$

$$\begin{aligned}
& + \frac{\operatorname{Ci}\left(\frac{(b-d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \cos\left(\frac{-ad+bc}{d}\right)}{d} \Bigg) \\
& + \frac{1}{2(b-d)} \left( d^2 c \left( \frac{\operatorname{Si}\left(\frac{(b-d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \sin\left(\frac{-ad+bc}{d}\right)}{d} \right. \right. \\
& + \frac{\operatorname{Ci}\left(\frac{(b-d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \cos\left(\frac{-ad+bc}{d}\right)}{d} \Bigg) - \frac{d \sin\left(\frac{(b+d)(dx+c)}{d} + \frac{ad-bc}{d}\right)}{2(b+d)} - \frac{1}{2(b+d)} \left( (ad \right. \\
& - bc) d \left( \frac{\operatorname{Si}\left(\frac{(b+d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \sin\left(\frac{-ad+bc}{d}\right)}{d} \right. \\
& + \frac{\operatorname{Ci}\left(\frac{(b+d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \cos\left(\frac{-ad+bc}{d}\right)}{d} \Bigg) \Bigg) \\
& + \frac{1}{2(b+d)} \left( a d^2 \left( \frac{\operatorname{Si}\left(\frac{(b+d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \sin\left(\frac{-ad+bc}{d}\right)}{d} \right. \right. \\
& + \frac{\operatorname{Ci}\left(\frac{(b+d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \cos\left(\frac{-ad+bc}{d}\right)}{d} \Bigg) \Bigg) \\
& + \frac{1}{2(b+d)} \left( d^2 c \left( \frac{\operatorname{Si}\left(\frac{(b+d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \sin\left(\frac{-ad+bc}{d}\right)}{d} \right. \right. \\
& + \frac{\operatorname{Ci}\left(\frac{(b+d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \cos\left(\frac{-ad+bc}{d}\right)}{d} \Bigg) \Bigg) \\
& - \frac{1}{2b} \left( d^2 \left( \frac{\operatorname{Si}\left(\frac{(b-d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \cos\left(\frac{-ad+bc}{d}\right)}{d} \right. \right.
\end{aligned}$$

$$\begin{aligned}
& - \frac{\text{Ci}\left(\frac{(b-d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \sin\left(\frac{-ad+bc}{d}\right)}{d} \Bigg) \\
& + \frac{1}{2b} \left( d^2 \left( \frac{\text{Si}\left(\frac{(b+d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \cos\left(\frac{-ad+bc}{d}\right)}{d} \right. \right. \\
& \left. \left. - \frac{\text{Ci}\left(\frac{(b+d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \sin\left(\frac{-ad+bc}{d}\right)}{d} \right) \right) \Bigg) \Bigg)
\end{aligned}$$

Problem 28: Unable to integrate problem.

$$\int x^2 \text{Ci}(d(a + b \ln(cx^n))) dx$$

Optimal(type 4, 131 leaves, 7 steps):

$$\frac{x^3 \text{Ci}(d(a + b \ln(cx^n)))}{3} - \frac{x^3 \text{Ei}\left(\frac{(3 - 1bdn)(a + b \ln(cx^n))}{bn}\right)}{6 e^{\frac{3a}{bn}} (cx^n)^{\frac{3}{n}}} - \frac{x^3 \text{Ei}\left(\frac{(3 + 1bdn)(a + b \ln(cx^n))}{bn}\right)}{6 e^{\frac{3a}{bn}} (cx^n)^{\frac{3}{n}}}$$

Result(type 8, 19 leaves):

$$\int x^2 \text{Ci}(d(a + b \ln(cx^n))) dx$$

Problem 29: Unable to integrate problem.

$$\int x \text{Ci}(d(a + b \ln(cx^n))) dx$$

Optimal(type 4, 131 leaves, 7 steps):

$$\frac{x^2 \text{Ci}(d(a + b \ln(cx^n)))}{2} - \frac{x^2 \text{Ei}\left(\frac{(2 - 1bdn)(a + b \ln(cx^n))}{bn}\right)}{4 e^{\frac{2a}{bn}} (cx^n)^{\frac{2}{n}}} - \frac{x^2 \text{Ei}\left(\frac{(2 + 1bdn)(a + b \ln(cx^n))}{bn}\right)}{4 e^{\frac{2a}{bn}} (cx^n)^{\frac{2}{n}}}$$

Result(type 8, 17 leaves):

$$\int x \text{Ci}(d(a + b \ln(cx^n))) dx$$

Problem 30: Unable to integrate problem.

$$\int \text{Ci}(d(a + b \ln(cx^n))) dx$$

Optimal(type 4, 118 leaves, 7 steps):

$$x \operatorname{Ci}(d(a + b \ln(cx^n))) - \frac{x \operatorname{Ei}\left(\frac{(1 - Ibdn)(a + b \ln(cx^n))}{bn}\right)}{2e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}} - \frac{x \operatorname{Ei}\left(\frac{(1 + Ibdn)(a + b \ln(cx^n))}{bn}\right)}{2e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}}$$

Result(type 8, 15 leaves):

$$\int \operatorname{Ci}(d(a + b \ln(cx^n))) dx$$

Problem 39: Result more than twice size of optimal antiderivative.

$$\int x \operatorname{Ci}(dx + c) \sin(bx + a) dx$$

Optimal(type 4, 351 leaves, 24 steps):

$$\begin{aligned} & -\frac{c \operatorname{Ci}\left(\frac{c(b-d)}{d} + (b-d)x\right) \cos\left(a - \frac{bc}{d}\right)}{2bd} - \frac{c \operatorname{Ci}\left(\frac{c(b+d)}{d} + (b+d)x\right) \cos\left(a - \frac{bc}{d}\right)}{2bd} - \frac{x \operatorname{Ci}(dx + c) \cos(bx + a)}{b} \\ & -\frac{\cos\left(a - \frac{bc}{d}\right) \operatorname{Si}\left(\frac{c(b-d)}{d} + (b-d)x\right)}{2b^2} - \frac{\cos\left(a - \frac{bc}{d}\right) \operatorname{Si}\left(\frac{c(b+d)}{d} + (b+d)x\right)}{2b^2} - \frac{\operatorname{Ci}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b^2} \\ & -\frac{\operatorname{Ci}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b^2} + \frac{c \operatorname{Si}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2bd} + \frac{c \operatorname{Si}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2bd} \\ & + \frac{\operatorname{Ci}(dx + c) \sin(bx + a)}{b^2} + \frac{\sin(a - c + (b-d)x)}{2b(b-d)} + \frac{\sin(a + c + (b+d)x)}{2b(b+d)} \end{aligned}$$

Result(type 4, 1207 leaves):

$$\begin{aligned} & \frac{1}{d} \left( \frac{1}{b} \left( \operatorname{Ci}(dx + c) \left( \frac{d \left( \sin\left(\frac{b(dx+c)}{d} + \frac{ad-bc}{d}\right) - \left(\frac{b(dx+c)}{d} + \frac{ad-bc}{d}\right) \cos\left(\frac{b(dx+c)}{d} + \frac{ad-bc}{d}\right) \right)}{b} \right. \right. \right. \\ & \left. \left. + \frac{d \cos\left(\frac{b(dx+c)}{d} + \frac{ad-bc}{d}\right) a}{b} \right) \right) - \frac{1}{b} \left( -\frac{d \sin\left(\frac{(b-d)(dx+c)}{d} + \frac{ad-bc}{d}\right)}{2(b-d)} - \frac{1}{2(b-d)} \left( (ad \right. \right. \\ & \left. \left. - bc) d \left( \frac{\operatorname{Si}\left(\frac{(b-d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \sin\left(\frac{-ad+bc}{d}\right)}{d} \right) \right) \end{aligned}$$



$$\begin{aligned}
& + \frac{\operatorname{Ci}\left(\frac{(b-d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \cos\left(\frac{-ad+bc}{d}\right)}{d} \Bigg) \\
& + \frac{1}{2(b-d)} \left( a d^2 \left( \frac{\operatorname{Si}\left(\frac{(b-d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \sin\left(\frac{-ad+bc}{d}\right)}{d} \right. \right. \\
& \left. \left. + \frac{\operatorname{Ci}\left(\frac{(b-d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \cos\left(\frac{-ad+bc}{d}\right)}{d} \right) \right) \\
& - \frac{1}{2(b-d)} \left( d^2 c \left( \frac{\operatorname{Si}\left(\frac{(b-d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \sin\left(\frac{-ad+bc}{d}\right)}{d} \right. \right. \\
& \left. \left. + \frac{\operatorname{Ci}\left(\frac{(b-d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \cos\left(\frac{-ad+bc}{d}\right)}{d} \right) \right) - \frac{d \sin\left(\frac{(b+d)(dx+c)}{d} + \frac{ad-bc}{d}\right)}{2(b+d)} - \frac{1}{2(b+d)} \left( (ad \right. \\
& \left. - bc) d \left( \frac{\operatorname{Si}\left(\frac{(b+d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \sin\left(\frac{-ad+bc}{d}\right)}{d} \right. \right. \\
& \left. \left. + \frac{\operatorname{Ci}\left(\frac{(b+d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \cos\left(\frac{-ad+bc}{d}\right)}{d} \right) \right) \\
& + \frac{1}{2(b+d)} \left( a d^2 \left( \frac{\operatorname{Si}\left(\frac{(b+d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \sin\left(\frac{-ad+bc}{d}\right)}{d} \right. \right. \\
& \left. \left. + \frac{\operatorname{Ci}\left(\frac{(b+d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \cos\left(\frac{-ad+bc}{d}\right)}{d} \right) \right) \\
& + \frac{1}{2(b+d)} \left( d^2 c \left( \frac{\operatorname{Si}\left(\frac{(b+d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \sin\left(\frac{-ad+bc}{d}\right)}{d} \right. \right.
\end{aligned}$$

$$\begin{aligned}
& + \frac{\text{Ci}\left(\frac{(b+d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \cos\left(\frac{-ad+bc}{d}\right)}{d} \Bigg) \Bigg) \\
& + \frac{1}{2b} \left( d^2 \left( \frac{\text{Si}\left(\frac{(b-d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \cos\left(\frac{-ad+bc}{d}\right)}{d} \right. \right. \\
& - \left. \left. \frac{\text{Ci}\left(\frac{(b-d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \sin\left(\frac{-ad+bc}{d}\right)}{d} \right) \right) \Bigg) \\
& + \frac{1}{2b} \left( d^2 \left( \frac{\text{Si}\left(\frac{(b+d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \cos\left(\frac{-ad+bc}{d}\right)}{d} \right. \right. \\
& - \left. \left. \frac{\text{Ci}\left(\frac{(b+d)(dx+c)}{d} + \frac{ad-bc}{d} + \frac{-ad+bc}{d}\right) \sin\left(\frac{-ad+bc}{d}\right)}{d} \right) \right) \Bigg) \Bigg) \Bigg)
\end{aligned}$$

Test results for the 40 problems in "8.5 Hyperbolic integral functions.txt"

Problem 1: Result unnecessarily involves higher level functions.

$$\int x^m \text{Shi}(bx) \, dx$$

Optimal(type 4, 72 leaves, 5 steps):

$$-\frac{x^m \Gamma(1+m, -bx)}{2b(1+m)(-bx)^m} - \frac{x^m \Gamma(1+m, bx)}{2b(1+m)(bx)^m} + \frac{x^{1+m} \text{Shi}(bx)}{1+m}$$

Result(type 5, 36 leaves):

$$\frac{bx^{2+m} \text{hypergeom}\left(\left[\frac{1}{2}, 1 + \frac{m}{2}\right], \left[\frac{3}{2}, \frac{3}{2}, 2 + \frac{m}{2}\right], \frac{b^2 x^2}{4}\right)}{2+m}$$

Problem 10: Unable to integrate problem.

$$\int \frac{\text{Shi}(bx+a)}{x^2} \, dx$$

Optimal(type 4, 46 leaves, 7 steps):

$$\frac{b \cosh(a) \text{Shi}(bx)}{a} - \frac{b \text{Shi}(bx+a)}{a} - \frac{\text{Shi}(bx+a)}{x} + \frac{b \text{Chi}(bx) \sinh(a)}{a}$$

Result(type 8, 12 leaves):

$$\int \frac{\text{Shi}(bx+a)}{x^2} dx$$

Problem 13: Unable to integrate problem.

$$\int \frac{\text{Shi}(bx) \sinh(bx)}{x^3} dx$$

Optimal(type 4, 84 leaves, 14 steps):

$$b^2 \text{Chi}(2bx) - \frac{b \cosh(bx) \text{Shi}(bx)}{2x} + \frac{b^2 \text{Shi}(bx)^2}{4} - \frac{b \cosh(bx) \sinh(bx)}{2x} - \frac{\text{Shi}(bx) \sinh(bx)}{2x^2} - \frac{\sinh(bx)^2}{4x^2} - \frac{b \sinh(2bx)}{4x}$$

Result(type 8, 14 leaves):

$$\int \frac{\text{Shi}(bx) \sinh(bx)}{x^3} dx$$

Problem 16: Unable to integrate problem.

$$\int \frac{\cosh(bx) \text{Shi}(bx)}{x^2} dx$$

Optimal(type 4, 40 leaves, 7 steps):

$$b \text{Chi}(2bx) - \frac{\cosh(bx) \text{Shi}(bx)}{x} + \frac{b \text{Shi}(bx)^2}{2} - \frac{\sinh(2bx)}{2x}$$

Result(type 8, 14 leaves):

$$\int \frac{\cosh(bx) \text{Shi}(bx)}{x^2} dx$$

Problem 20: Unable to integrate problem.

$$\int \text{Shi}(dx+c) \sinh(bx+a) dx$$

Optimal(type 4, 145 leaves, 9 steps):

$$\frac{\cosh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{c(b-d)}{d} + (b-d)x\right)}{2b} + \frac{\cosh(bx+a) \text{Shi}(dx+c)}{b} - \frac{\cosh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{c(b+d)}{d} + (b+d)x\right)}{2b}$$

$$+ \frac{\text{Chi}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sinh\left(a - \frac{bc}{d}\right)}{2b} - \frac{\text{Chi}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sinh\left(a - \frac{bc}{d}\right)}{2b}$$

Result(type 8, 15 leaves):

$$\int \text{Shi}(dx+c) \sinh(bx+a) dx$$

Problem 22: Unable to integrate problem.

$$\int x \cosh(bx + a) \operatorname{Shi}(dx + c) dx$$

Optimal(type 4, 351 leaves, 24 steps):

$$\begin{aligned} & -\frac{c \operatorname{Chi}\left(\frac{c(b-d)}{d} + (b-d)x\right) \cosh\left(a - \frac{bc}{d}\right)}{2bd} + \frac{c \operatorname{Chi}\left(\frac{c(b+d)}{d} + (b+d)x\right) \cosh\left(a - \frac{bc}{d}\right)}{2bd} - \frac{\cosh\left(a - \frac{bc}{d}\right) \operatorname{Shi}\left(\frac{c(b-d)}{d} + (b-d)x\right)}{2b^2} \\ & - \frac{\cosh(bx + a) \operatorname{Shi}(dx + c)}{b^2} + \frac{\cosh\left(a - \frac{bc}{d}\right) \operatorname{Shi}\left(\frac{c(b+d)}{d} + (b+d)x\right)}{2b^2} - \frac{\operatorname{Chi}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sinh\left(a - \frac{bc}{d}\right)}{2b^2} \\ & + \frac{\operatorname{Chi}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sinh\left(a - \frac{bc}{d}\right)}{2b^2} - \frac{c \operatorname{Shi}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sinh\left(a - \frac{bc}{d}\right)}{2bd} + \frac{c \operatorname{Shi}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sinh\left(a - \frac{bc}{d}\right)}{2bd} \\ & + \frac{x \operatorname{Shi}(dx + c) \sinh(bx + a)}{b} + \frac{\sinh(a - c + (b-d)x)}{2b(b-d)} - \frac{\sinh(a + c + (b+d)x)}{2b(b+d)} \end{aligned}$$

Result(type 8, 16 leaves):

$$\int x \cosh(bx + a) \operatorname{Shi}(dx + c) dx$$

Problem 26: Unable to integrate problem.

$$\int \frac{\operatorname{Chi}(bx + a)}{x^2} dx$$

Optimal(type 4, 46 leaves, 7 steps):

$$-\frac{b \operatorname{Chi}(bx + a)}{a} - \frac{\operatorname{Chi}(bx + a)}{x} + \frac{b \operatorname{Chi}(bx) \cosh(a)}{a} + \frac{b \operatorname{Shi}(bx) \sinh(a)}{a}$$

Result(type 8, 12 leaves):

$$\int \frac{\operatorname{Chi}(bx + a)}{x^2} dx$$

Problem 27: Unable to integrate problem.

$$\int \frac{\operatorname{Chi}(bx + a)}{x^3} dx$$

Optimal(type 4, 97 leaves, 11 steps):

$$\frac{b^2 \operatorname{Chi}(bx + a)}{2a^2} - \frac{\operatorname{Chi}(bx + a)}{2x^2} - \frac{b^2 \operatorname{Chi}(bx) \cosh(a)}{2a^2} - \frac{b \cosh(bx + a)}{2ax} + \frac{b^2 \cosh(a) \operatorname{Shi}(bx)}{2a} + \frac{b^2 \operatorname{Chi}(bx) \sinh(a)}{2a} - \frac{b^2 \operatorname{Shi}(bx) \sinh(a)}{2a^2}$$

Result(type 8, 12 leaves):

$$\int \frac{\operatorname{Chi}(bx + a)}{x^3} dx$$

Problem 28: Unable to integrate problem.

$$\int x^2 \operatorname{Chi}(d(a + b \ln(cx^n))) dx$$

Optimal(type 4, 128 leaves, 7 steps):

$$\frac{x^3 \operatorname{Chi}(d(a + b \ln(cx^n)))}{3} - \frac{x^3 \operatorname{Ei}\left(\frac{(-bdn + 3)(a + b \ln(cx^n))}{bn}\right)}{6 e^{\frac{3a}{bn}} (cx^n)^{\frac{3}{n}}} - \frac{x^3 \operatorname{Ei}\left(\frac{(bdn + 3)(a + b \ln(cx^n))}{bn}\right)}{6 e^{\frac{3a}{bn}} (cx^n)^{\frac{3}{n}}}$$

Result(type 8, 19 leaves):

$$\int x^2 \operatorname{Chi}(d(a + b \ln(cx^n))) dx$$

Problem 29: Unable to integrate problem.

$$\int x \operatorname{Chi}(d(a + b \ln(cx^n))) dx$$

Optimal(type 4, 128 leaves, 7 steps):

$$\frac{x^2 \operatorname{Chi}(d(a + b \ln(cx^n)))}{2} - \frac{x^2 \operatorname{Ei}\left(\frac{(-bdn + 2)(a + b \ln(cx^n))}{bn}\right)}{4 e^{\frac{2a}{bn}} (cx^n)^{\frac{2}{n}}} - \frac{x^2 \operatorname{Ei}\left(\frac{(bdn + 2)(a + b \ln(cx^n))}{bn}\right)}{4 e^{\frac{2a}{bn}} (cx^n)^{\frac{2}{n}}}$$

Result(type 8, 17 leaves):

$$\int x \operatorname{Chi}(d(a + b \ln(cx^n))) dx$$

Problem 30: Unable to integrate problem.

$$\int \operatorname{Chi}(d(a + b \ln(cx^n))) dx$$

Optimal(type 4, 115 leaves, 7 steps):

$$x \operatorname{Chi}(d(a + b \ln(cx^n))) - \frac{x \operatorname{Ei}\left(\frac{(-bdn + 1)(a + b \ln(cx^n))}{bn}\right)}{2 e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}}} - \frac{x \operatorname{Ei}\left(\frac{(bdn + 1)(a + b \ln(cx^n))}{bn}\right)}{2 e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}}}$$

Result(type 8, 15 leaves):

$$\int \operatorname{Chi}(d(a + b \ln(cx^n))) dx$$

Problem 39: Unable to integrate problem.

$$\int x \operatorname{Chi}(dx + c) \sinh(bx + a) dx$$

Optimal(type 4, 351 leaves, 24 steps):

$$\begin{aligned} & \frac{c \operatorname{Chi}\left(\frac{c(b-d)}{d} + (b-d)x\right) \cosh\left(a - \frac{bc}{d}\right)}{2bd} + \frac{c \operatorname{Chi}\left(\frac{c(b+d)}{d} + (b+d)x\right) \cosh\left(a - \frac{bc}{d}\right)}{2bd} + \frac{x \operatorname{Chi}(dx+c) \cosh(bx+a)}{b} \\ & + \frac{\cosh\left(a - \frac{bc}{d}\right) \operatorname{Shi}\left(\frac{c(b-d)}{d} + (b-d)x\right)}{2b^2} + \frac{\cosh\left(a - \frac{bc}{d}\right) \operatorname{Shi}\left(\frac{c(b+d)}{d} + (b+d)x\right)}{2b^2} + \frac{\operatorname{Chi}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sinh\left(a - \frac{bc}{d}\right)}{2b^2} \\ & + \frac{\operatorname{Chi}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sinh\left(a - \frac{bc}{d}\right)}{2b^2} + \frac{c \operatorname{Shi}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sinh\left(a - \frac{bc}{d}\right)}{2bd} + \frac{c \operatorname{Shi}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sinh\left(a - \frac{bc}{d}\right)}{2bd} \\ & - \frac{\operatorname{Chi}(dx+c) \sinh(bx+a)}{b^2} - \frac{\sinh(a-c+(b-d)x)}{2b(b-d)} - \frac{\sinh(a+c+(b+d)x)}{2b(b+d)} \end{aligned}$$

Result(type 8, 16 leaves):

$$\int x \operatorname{Chi}(dx+c) \sinh(bx+a) dx$$

Test results for the 63 problems in "8.6 Gamma functions.txt"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int x^{100} \operatorname{Ei}_1(ax) dx$$

Optimal(type 4, 21 leaves, 1 step):

$$\frac{x^{101} \operatorname{Ei}_1(ax)}{101} - \frac{\Gamma(101, ax)}{101 a^{101}}$$

Result(type 4, 1321 leaves):

$$\begin{aligned} & \frac{1}{a^{101}} \left( \right. \\ & - 1 / \\ & 101 ( \\ & 933262154439441526816992388562667004907159682643816214685929638952175999932299156089414639761565182862536979208272237582511852109168 \backslash \\ & 64000000000000000000000000000000000000 e^{-ax}) \\ & \left. - 1 / \right) \end{aligned}$$

101 (

15041736779921559705429150976957764193716230611477699300843098108055085095218424562720561438764952515238928045834240000000000000000\

000  $a^{24} x^{24} e^{-ax}$ )

- 1 /

101 (

83030387025167009573968913392806858349313592975356900140653901556464069725605703586217499141982537884118882813005004800000000000000\

00000  $a^{22} x^{22} e^{-ax}$ )

- 1 /

101 (

383600388056271584231736379874767685573828799546148878649821025190864002132298350568324846035959325024629238596083122176000000000000\

00000000  $a^{20} x^{20} e^{-ax}$ )

- 1 /

101 (

145768147461383202008059824352411720518054943827536573886931989572528320810273373215963441493664543509359110666511586426880000000000\

0000000000  $a^{18} x^{18} e^{-ax}$ )

- 1 /

101 (

446050531231832598144663062518379864785248128112261916094011888091936661679436522040848130970613503138638878639525454466252800000000\

0000000000000  $a^{16} x^{16} e^{-ax}$ )

$$\frac{154474566048065858941771694797159985079379545761219641534137533399350462184007117168653107200000000000000 a^{43} x^{43} e^{-ax}}{101}$$

$$\frac{278981066282806941248839680803670933053359459644762672610652385319226934704316853606587511603200000000000000 a^{41} x^{41} e^{-ax}}{101}$$

$$\frac{4575289487038033836480970765180203302075095138174107830814699119235321729150796399148035190292480000000000000000 a^{39} x^{39} e^{-ax}}{101}$$

$$\frac{6780579019790366145664798673997061293675290994774027805267384094706746802601480263537388152013455360000000000000000 a^{37} x^{37} e^{-ax}}{101}$$

$$\frac{9031731254360767706025511833764085643175487605039005036616155614149386741065171711031801018481922539520000000000000000 a^{35} x^{35} e^{-ax}}{101}$$

$$\frac{100 a^{99} x^{99} e^{-ax}}{101} - \frac{970200 a^{97} x^{97} e^{-ax}}{101} - \frac{9034502400 a^{95} x^{95} e^{-ax}}{101} - \frac{9900 a^{98} x^{98} e^{-ax}}{101} - \frac{94109400 a^{96} x^{96} e^{-ax}}{101}$$

- 1 /

101 (

107052127495639823554719135004411167548459550746942859862562853142064798803064765289803551432947240753273330873486109071900672000000\

0000000000000000  $a^{14} x^{14} e^{-ax}$ )

- 1 /

101 (

194834872042064478869588825708028324938196382359436004949864392718557933821577872827442463607963978170957462189744718510859223040000\

0000000000000000  $a^{12} x^{12} e^{-ax}$ )

- 1 /



101 (

257182031095525112107857249934597388918419224714455526533820998388496472644482792132224051962512451185663850090463028434334174412800\

00000000000000000000  $a^{10} x^{10} e^{-ax}$ )

- 1 /

101 (

231463827985972600897071524941137650026577302243009973880438898549646825380034512919001646766261206067097465081416725590900756971520\

00000000000000000000  $a^8 x^8 e^{-ax}$ )

- 1 /

101 (

129619743672144656502360053967037084014883289256085585373045783187802222212819327234640922189106275397574580445593366330904423904051\

20000000000000000000  $a^6 x^6 e^{-ax}$ )

- 1 /

101 (

388859231016433969507080161901111252044649867768256756119137349563406666638457981703922766567318826192723741336780098992713271712153\

60000000000000000000000000000000  $a^4 x^4 e^{-ax}$ ) -  $\frac{858277728000 a^{94} x^{94} e^{-ax}}{101}$  -  $\frac{7503063898176000 a^{92} x^{92} e^{-ax}}{101}$  -  $\frac{62815650955529472000 a^{90} x^{90} e^{-ax}}{101}$   
-  $\frac{503153364153791070720000 a^{88} x^{88} e^{-ax}}{101}$  -  $\frac{3852142155961424437432320000 a^{86} x^{86} e^{-ax}}{101}$  -  $\frac{28159159160078012637630259200000 a^{84} x^{84} e^{-ax}}{101}$   
-  $\frac{196325657664063904109558167142400000 a^{82} x^{82} e^{-ax}}{101}$  -  $\frac{1303995018204712451095685346159820800000 a^{80} x^{80} e^{-ax}}{101}$

$$- \frac{8241248515053782690924731387730067456000000 a^{78} x^{78} e^{-ax}}{101} - \frac{49496938581413018841693936714706785140736000000 a^{76} x^{76} e^{-ax}}{101}$$

$- 1 /$

101 (

466631077219720763408496194281333502453579841321908107342964819476087999966149578044707319880782591431268489604136118791255926054584 \

3200000000000000000000000000000000  $a^2 x^2 e^{-ax}$ )

$- 1 /$

101 (

933262154439441526816992388562667004907159682643816214685929638952175999932299156089414639761565182862536979208272237582511852109168 \

6400000000000000000000000000000000  $a x e^{-ax}$ )

$- 1 /$

101 (

60166947119686238821716603907831056774864922445910797203372392432220340380873698250882245755059810060955712183336960000000000000000 \

0  $a^{25} x^{25} e^{-ax}$ )

$- 1 /$

101 (

361001682718117432930299623446986340649189534675464783220234354593322042285242189505293474530358860365734273100021760000000000000000 \

0000  $a^{23} x^{23} e^{-ax}$ )

- 1 /

101 (

18266685145536742106273160946417508836848990454578518030943858342422095339633254788967849811236158334506154218861101056000000000000\

0000000  $a^{21} x^{21} e^{-ax}$ )

- 1 /

101 (

767200776112543168463472759749535371147657599092297757299642050381728004264596701136649692071918650049258477192166244352000000000000\

000000000  $a^{19} x^{19} e^{-ax}$ )

- 1 /

101 (

262382665430489763614507683834341096932498898889565832996477581230550977458492071788734194688596178316846399199720855568384000000000\

000000000000  $a^{17} x^{17} e^{-ax}$ )

- 1 /

101 (

155543692406573587802832064760444500817859947107302702447654939825362666655383192681569106626927530477089496534712039597085308684861\

44000000000000000000000000  $a^3 x^3 e^{-ax}$ )

- 1 /

$$101 (107477601926893135701703590821792619153788302499964159935732251808377702218675543361278432119934878220288000000000000000000 a^{33} x^{33} e^{-ax})$$

$$- 1 /$$

$$101 (1134963476347991513009989919078130058264004474399621528921332579096468535429213737895100243186512314006241280000000000000000000 a^{31} x^{31} e^{-ax})$$

$$- 1 /$$

$$101 (1055516033003632107099290624742660954185524161191648021896839298559715737949168776242443226163456452025804390400000000000000000000 a^{29} x^{29} e^{-ax})$$

$$- 1 /$$

$$101 (857079018798949270964623987291040694798645618887618193780233510430489179214725046308863899644726639044953165004800000000000000000000 a^{27} x^{27} e^{-ax}) + \frac{a^{101} x^{101} \operatorname{Ei}_1(ax)}{101} - \frac{a^{100} x^{100} e^{-ax}}{101}$$

$$- \frac{25088142373224354738959755093789126786598576680663902879489321150414963169625476975088336162449784832000000000000000000 a^{36} x^{36} e^{-ax}}{101}$$

$$- 1 /$$

$$101 (316110593902626869710892914181742997511142066176365176281565446495228535937281009886113035646867288883200000000000000000000 a^{34} x^{34} e^{-ax}) - \frac{1524080034635720828362134682957222503982458470400000000 a^{72} x^{72} e^{-ax}}{101}$$

$$\frac{7791097137057804874587232499277321440358327700684800000000 a^{70} x^{70} e^{-ax}}{101}$$

$$\frac{37630999171989197544256332971509462556930722794307584000000000 a^{68} x^{68} e^{-ax}}{101}$$

$$\frac{171446832227582784011631853018197111409376373050865352704000000000 a^{66} x^{66} e^{-ax}}{101}$$

$$\frac{735506910256330143409900649448065607946224640388212363100160000000000 a^{64} x^{64} e^{-ax}}{101}$$

$$\frac{2965563862153523138228719418574600531239177750045272248019845120000000000 a^{62} x^{62} e^{-ax}}{101}$$

$$\frac{11215762526664624508781016841049139209146570250671219642011054243840000000000 a^{60} x^{60} e^{-ax}}{101}$$

$$\frac{39703799344392770761084799617313952800378858687376117532719132023193600000000000 a^{58} x^{58} e^{-ax}}{101}$$

$$\frac{131260760632562500136146347534839927958052506820465444563169450468678041600000000000 a^{56} x^{56} e^{-ax}}{101}$$

$$\frac{404283142748292500419330750407306978110801721007033569254561907443528368128000000000000 a^{54} x^{54} e^{-ax}}{101}$$

$$\frac{1157058354545613136200124607665712571353114525522130075206556179103378189582336000000000000 a^{52} x^{52} e^{-ax}}{101}$$

$$\frac{3068518756254966037202730459529469739228459721684688959447786986982158958772355072000000000000 a^{50} x^{50} e^{-ax}}{101}$$

$$\frac{751787095282466679114668962584720086110972631812748795064707811810628944899226992640000000000000 a^{48} x^{48} e^{-ax}}{101}$$

$$\frac{16960316869572448280826931795911285142663542573695612816659808234447788996926560953958400000000000000 a^{46} x^{46} e^{-ax}}{101}$$

$$\frac{35107855920014967941311748817536360245313533127549918530485803045306923223637981174693888000000000000000 a^{44} x^{44} e^{-ax}}{101}$$

$$\frac{66424063400668319344961828762778793584133204677324445859679139361720698739123060382520836096000000000000000 a^{42} x^{42} e^{-ax}}{101}$$

$$\frac{1143822371759508459120242691295050825518773784543526957703674779808830432287699099787008797573120000000000000000 a^{40} x^{40} e^{-ax}}{101}$$

$$\frac{178436289994483319622757859842027928780928710388790205401773265650177547436881059566773372421406720000000000000000 a^{38} x^{38} e^{-ax}}{101}$$

- 1 /

101 (

713680849970932157031460900029407783656397004979619065750419020947098658687098435265357009552981605021822205823240727146004480000000\

00000000000000  $a^{15} x^{15} e^{-ax}$ )

- 1 /

101 (

149872978493895752976606789006175634567843371045720003807587994398890718324290671405724972006126137054582663222880552700660940800000\

000000000000000000  $a^{13} x^{13} e^{-ax}$ )

- 1 /

101 (

233801846450477374643506590849633989925835658831323205939837271262269520585893447392930956329556773805148954627693662213031067648000\

00000000000000000000  $a^{11} x^{11} e^{-ax}$ )

- 1 /

101 (

257182031095525112107857249934597388918419224714455526533820998388496472644482792132224051962512451185663850090463028434334174412800\

0000000000000000000000  $a^9 x^9 e^{-ax}$ )

- 1 /

101 (

185171062388778080717657219952910120021261841794407979104351118839717460304027610335201317413008964853677972065133380472720605577216\

000000000000000000000000  $a^7 x^7 e^{-ax}$ )

- 1 /

101 (

777718462032867939014160323802222504089299735536513512238274699126813333276915963407845533134637652385447482673560197985426543424307\

$$\begin{aligned} & 200000000000000000000000000000 a^5 x^5 e^{-ax} - \frac{80678106432000 a^{93} x^{93} e^{-ax}}{101} - \frac{690281878632192000 a^{91} x^{91} e^{-ax}}{101} - \frac{5653408585997652480000 a^{89} x^{89} e^{-ax}}{101} \\ & - \frac{44277496045533614223360000 a^{87} x^{87} e^{-ax}}{101} - \frac{331284225412682501619179520000 a^{85} x^{85} e^{-ax}}{101} - \frac{2365369369446553061560941772800000 a^{83} x^{83} e^{-ax}}{101} \\ & - \frac{16098703928453240136983769705676800000 a^{81} x^{81} e^{-ax}}{101} - \frac{104319601456376996087654827692785664000000 a^{79} x^{79} e^{-ax}}{101} \\ & - \frac{642817384174195049892129048242945261568000000 a^{77} x^{77} e^{-ax}}{101} - \frac{3761767332187389431968739190317715670695936000000 a^{75} x^{75} e^{-ax}}{101} \\ & - \frac{20877808693640011347426502506263321972362444800000000 a^{73} x^{73} e^{-ax}}{101} \\ & - \frac{109733762493771899642073697172920020286737009868800000000 a^{71} x^{71} e^{-ax}}{101} \\ & - \frac{545376799594046341221106274949412500825082939047936000000000 a^{69} x^{69} e^{-ax}}{101} \\ & - \frac{2558907943695265433009430642062643453871289150012915712000000000 a^{67} x^{67} e^{-ax}}{101} \\ & - \frac{11315490927020463744767702299201009353018840621357113278464000000000 a^{65} x^{65} e^{-ax}}{101} \\ & - \frac{47072442256405129178233641564676198908558376984845591238410240000000000 a^{63} x^{63} e^{-ax}}{101} \\ & - \frac{183864959453518434570180603951625232936829020502806879377230397440000000000 a^{61} x^{61} e^{-ax}}{101} \\ & - \frac{672945751599877470526861010462948352548794215040273178520663254630400000000000 a^{59} x^{59} e^{-ax}}{101} \\ & - \frac{2302820361974780704142918377804209262421973803867814816897709657345228800000000000 a^{57} x^{57} e^{-ax}}{101} \\ & - \frac{7350602595423500007624195461951035965650940381946064895537489226245970329600000000000 a^{55} x^{55} e^{-ax}}{101} \\ & - \frac{21831289708407795022643860521994576817983292934379812739746343001950531878912000000000000 a^{53} x^{53} e^{-ax}}{101} \\ & - \frac{60167034436371883082406479598617053710361955327150763910740921313375665858281472000000000000 a^{51} x^{51} e^{-ax}}{101} \\ & - \frac{153425937812748301860136522976473486961422986084234447972389349349107947938617753600000000000000 a^{49} x^{49} e^{-ax}}{101} \end{aligned}$$

$$\frac{36085780573558400597504110204066564133326686327011942163105974966910189355162895646720000000000000 a^{47} x^{47} e^{-ax}}{101} - \frac{780174576000332620918038862611919116562522958389998189566351178784598293858621803882086400000000000000 a^{45} x^{45} e^{-ax}}{101}$$

- 1 /

$$101 (35467608635874734781562184971191564320750139824988172778791643096764641732162929309221882599578509812695040000000000000000 a^{32} x^{32} e^{-ax})$$

- 1 /

$$101 (351838677667877369033096874914220318061841387063882673965613099519905245983056258747481075387818817341934796800000000000000000 a^{30} x^{30} e^{-ax})$$

- 1 /

$$101 (30609964957105331105879428117537167671380200674557792635008339658231756400525894511030853558740237108748327321600000000000000000 a^{28} x^{28} e^{-ax})$$

- 1 /

$$101 (23141133507571630316044847656858098759563431709965691232066304781623207838797576250339325290407619254213735455129600000000000000000 a^{26} x^{26} e^{-ax} - \frac{282132549914054207397655439273828675302195200000000 a^{74} x^{74} e^{-ax}}{101} )$$

Problem 3: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Ei}_1(ax)}{x^3} dx$$

Optimal (type 4, 21 leaves, 1 step):



$$\frac{\text{Ei}_3(ax)}{2x^2} - \frac{\text{Ei}_1(ax)}{2x^2}$$

Result(type 4, 51 leaves):

$$a^2 \left( -\frac{\text{Ei}_1(ax)}{2a^2x^2} + \frac{e^{-ax}}{4a^2x^2} - \frac{e^{-ax}}{4ax} + \frac{\text{Ei}_1(ax)}{4} \right)$$

Problem 7: Maple result simpler than optimal antiderivative, IF it can be verified!

$$\int x^{100} e^{-ax} (ax+1) dx$$

Optimal(type 4, 26 leaves, 1 step):

$$\frac{x^{101} e^{-ax} (ax+1)}{101} - \frac{\Gamma(103, ax)}{101 a^{101}}$$

Result(type 3, 815 leaves):

$$-\frac{1}{a^{101}} \left( (x^{101} a^{101} + 102x^{100} a^{100} + 10200x^{99} a^{99} + 1009800x^{98} a^{98} + 98960400x^{97} a^{97} + 9599158800x^{96} a^{96} + 921519244800x^{95} a^{95} + 87544328256000x^{94} a^{94} \right. \\ + 8229166856064000x^{93} a^{93} + 765312517613952000x^{92} a^{92} + 70408751620483584000x^{91} a^{91} + 6407196397464006144000x^{90} a^{90} \\ + 576647675771760552960000x^{89} a^{89} + 51321643143686689213440000x^{88} a^{88} + 4516304596644428650782720000x^{87} a^{87} \\ + 392918499908065292618096640000x^{86} a^{86} + 33790990992093615165156311040000x^{85} a^{85} + 2872234234327957289038286438400000x^{84} a^{84} \\ + 241267675683548412279216060825600000x^{83} a^{83} + 20025217081734518219174933048524800000x^{82} a^{82} \\ + 1642067800702230493972344509979033600000x^{81} a^{81} + 133007491856880670011759905308301721600000x^{80} a^{80} \\ + 10640599348550453600940792424664137728000000x^{79} a^{79} + 840607348535485834474322601548466880512000000x^{78} a^{78} \\ + 65567373185767895088997162920780416679936000000x^{77} a^{77} + 5048687735304127921852781544900092084355072000000x^{76} a^{76} \\ + 383700267883113722060811397412406998410985472000000x^{75} a^{75} + 28777520091233529154560854805930524880823910400000000x^{74} a^{74} \\ + 2129536486751281157437503255638858841180969369600000000x^{73} a^{73} + 155456163532843524492937737661636695406210763980800000000x^{72} a^{72} \\ + 11192843774364733763491517111637842069247175006617600000000x^{71} a^{71} \\ + 794691907979896097207897714926286786916549425469849600000000x^{70} a^{70} \\ + 55628433558592726804552840044840075084158459782889472000000000x^{69} a^{69} \\ + 3838361915542898149514145963093965180806933725019373568000000000x^{68} a^{68} \\ + 261008610256917074166961925490389632294871493301317402624000000000x^{67} a^{67} \\ + 17487576887213443969186449007856105363756390051188265975808000000000x^{66} a^{66} \\ + 1154180074556087301966305634518502954007921743378425554403328000000000x^{65} a^{65} \\ + 75021704846145674627809866243702692010514913319597661036216320000000000x^{64} a^{64} \\ + 4801389110153323176179831439596972288672954452454250306317844480000000000x^{63} a^{63} \\ + 302487513939659360099329380694609254186396130504617769298024202240000000000x^{62} a^{62} \\ + 18754225864258880326158421603065773759556560091286301696477500538880000000000x^{61} a^{61} \left. \right)$$

$$\begin{aligned}
& + 114400777719791699895663717787012199332950165568464403485127532871680000000000 x^{60} a^{60} \\
& + 68640466663187501993739823067220731959977009934107864209107651972300800000000000 x^{59} a^{59} \\
& + 4049787533128062617630649560966023185638643586112363988337351466365747200000000000 x^{58} a^{58} \\
& + 234887676921427631822577674536029344767041327994517111323566385049213337600000000000 x^{57} a^{57} \\
& + 13388597584521375013886927448553672651721355695687475345443283947805160243200000000000 x^{56} a^{56} \\
& + 749761464733197000777667937119005668496395918958498619344823901077088973619200000000000 x^{55} a^{55} \\
& + 4123688056032583504277173654154531176730177554271742406396531455923989354905600000000000 x^{54} a^{54} \\
& + 222679155025759509230967377324344683543429587930674089945412698619895425164902400000000000 x^{53} a^{53} \\
& + 11801995216365253989241270998190268227801768160325726767106873026854457533739827200000000000 x^{52} a^{52} \\
& + 613703751250993207440546091905893947845691944336937791889557397396431791754471014400000000000 x^{51} a^{51} \\
& + 31298891313800653579467850687200591340130289161183827386367427267218021379478021734400000000000 x^{50} a^{50} \\
& + 1564944565690032678973392534360029567006514458059191369318371363360901068973901086720000000000000 x^{49} a^{49} \\
& + 76682283718811601269696234183641448783319208444900377096600196804684152379721153249280000000000000 x^{48} a^{48} \\
& + 3680749618502956860945419240814789541599322005355218100636809446624839314226615355965440000000000000 x^{47} a^{47} \\
& + 172995232069638972464434704318295108455168134251695250729930043991367447768650921730375680000000000000 x^{46} a^{46} \\
& + 79577806752033927333639963986415749889377341755779815335767820236029025973579423995972812800000000000000 x^{45} a^{45} \\
& + 358100130384152673001379837938870874502198037901009169010955191062130616881107407981877657600000000000000 x^{44} a^{44} \\
& + 15756405736902717612060712869310318478096713667644403436482028406733747142768725951202616934400000000000000 x^{43} a^{43} \\
& + 6775254466868168573186106533803436945581586877087093477687272214895511271390552159017125281792000000000000000 x^{42} a^{42} \\
& + 284560687608463080073816474419744351714426648837657926062865433025611473398403190678719261835264000000000000000 x^{41} a^{41} \\
& + 11666988191946986283026475451209518420291492602343974968577482754050070409334530817827489735245824000000000000000 x^{40} a^{40} \\
& + 46667952767787945132105901804838073681165970409375899874309931016200281637338123271309958940983296000000000000000 x^{39} a^{39} \\
& + 1820050157943729860152130170388684873565472845965660095098087309631810983856186807581088398698348544000000000000000 x^{38} a^{38} \\
& + 69161906001861734685780946474770025195487968146695083613727317766008817386535098688081359150537244672000000000000000 x^{37} a^{37} \\
& + 2558990522068884183373895019566490932233054821427718093707910757342326243301798651459010288569878052864000000000000000 x^{36} a^{36} \\
& + 92123658794479830601460220704393673560389973571397851373484787264323744758864751452524370388515609903104000000000000000 x^{35} a^{35} \\
& + 322432805780679407105110772465377857461364907499892479807196755425133106656026630083835296359804634660864000000000000000 x^{34} a^{34} \\
& + 10962715396543099841573766263822847153686406854996344313444689684454525626304905422850400076233357578469376000000000000000 x^{33} a^{33} \\
& + 361769608085922294771934286706153956071651426214879362343674759586999345668061878954063202515700800089489408000000000000000 x^{32} a^{32} \\
& + 11576627458749513432701897174596926594292845638876139594997592306783979061377980126530022480502425602863661056000000000000000 x^{31} a^{31} \\
& + 358875451221234916413758812412504724423078214805160327444925361510303350902717383922430696895575193688773492736000000000000000 x^{30} a^{30}
\end{aligned}$$

+ 10766263536637047492412764372375141732692346444154809823347760845309100527081521517672920906867255810663204782080000000000000000\  
 $x^{29} a^{29}$   
+ 312221642562474377279970166798879110248078046880489484877085064513963915285364124012514706299150418509232938680320000000000000000\  
 $0 x^{28} a^{28}$   
+ 8742205991749282563839164670368615086946185312653705576558381806390989627990195472350411776376211718258522283048960000000000000000\  
 $00 x^{27} a^{27}$   
+ 236039561777230629223657446099952607347547003441650050567076308772556719955735277753461117962157716392980101642321920000000000000000\  
 $0000 x^{26} a^{26}$   
+ 613702860620799635981509359859876779103622208948290131474398402808647471884911722158998906701610062621748264270036992000000000000000\  
 $00000 x^{25} a^{25}$   
+ 1534257151551999089953773399649691947759055522370725328685996007021618679712279305397497266754025156554370660675092480000000000000000\  
 $0000000 x^{24} a^{24}$   
+ 3682217163724797815889056159159260674621733253689740788846390416851884831309470332953993440209660375730489585620221952000000000000000\  
 $00000000 x^{23} a^{23}$   
+ 8469099476567034976544829166066299551629986483486403814346697958759335112011781765794184912482218864180126046926510489600000000000000\  
 $000000000 x^{22} a^{22}$   
+ 18632018848447476948398624165345859013585970263670088391562735509270537246425919884747206807460881501196277303238323077120000000000\  
 $00000000000 x^{21} a^{21}$   
+ 39127239581739701591637110747226303928530537553707185622281744569468128217494431757969134295667851152512182336800478461952000000000\  
 $000000000000 x^{20} a^{20}$   
+ 78254479163479403183274221494452607857061075107414371244563489138936256434988863515938268591335702305024364673600956923904000000000\  
 $0000000000000 x^{19} a^{19}$   
+ 14868351041061086604822102083945995492841604270408730536467062936397888722647884068028271032353783437954629287984181815541760000000\  
 $000000000000000 x^{18} a^{18}$   
+ 26763031873909955888679783751102791887114887686735714965640713285516199700766191322450887858236810188318332718371527267975168000000\  
 $0000000000000000 x^{17} a^{17}$   
+ 4549715418564692501075563237687474620809530906745071544158921258537753949130252524816650935900257732014116562123159635555778560000\  
 $00000000000000000 x^{16} a^{16}$   
+ 7279544669703508001720901180299959393295249450792114470654274013660406318608404039706641497440412371222586499397055416889245696000\  
 $000000000000000000 x^{15} a^{15}$   
+ 1091931700455526200258135177044993908994287417618817170598141102049060947791260605955996224616061855683387974909558312533386854400\  
 $0000000000000000000 x^{14} a^{14}$   
+ 1528704380637736680361389247862991472592002384666344038837397542868685326907764848338394714462486597956743164873381637546741596160\  
 $00000000000000000000 x^{13} a^{13}$



$$\frac{\left(-\frac{4}{3} + \gamma + \ln(x) + \ln(a)\right)x^3 a^3}{3} + \frac{4a^3 x^3}{9} + \frac{1}{3} - \frac{(-8a^2 x^2 + 8ax + 8)e^{-ax}}{24} + \frac{a^3 x^3 (-\gamma - \ln(ax) - \text{Ei}_1(ax))}{3}$$

Problem 13: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Ei}_2(ax)}{ax^3} dx$$

Optimal(type 4, 26 leaves, 1 step):

$$\frac{\text{Ei}_3(ax)}{ax^2} - \frac{\text{Ei}_2(ax)}{ax^2}$$

Result(type 4, 105 leaves):

$$a \left( -\frac{1}{2a^2 x^2} - \frac{\gamma + \ln(x) + \ln(a)}{xa} - \frac{\gamma}{2} + \frac{5}{4} - \frac{\ln(x)}{2} - \frac{\ln(a)}{2} + \frac{-15a^2 x^2 + 6}{12a^2 x^2} - \frac{(6ax + 6)e^{-ax}}{12a^2 x^2} - \frac{(6ax + 12)(-\gamma - \ln(ax) - \text{Ei}_1(ax))}{12ax} \right)$$

Problem 14: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Ei}_2(ax)}{ax^5} dx$$

Optimal(type 4, 27 leaves, 1 step):

$$\frac{\text{Ei}_5(ax)}{3ax^4} - \frac{\text{Ei}_2(ax)}{3ax^4}$$

Result(type 4, 164 leaves):

$$a^3 \left( -\frac{1}{4a^4 x^4} - \frac{-\frac{2}{3} + \gamma + \ln(x) + \ln(a)}{3x^3 a^3} + \frac{1}{4a^2 x^2} - \frac{1}{12ax} + \frac{29}{864} - \frac{\gamma}{72} - \frac{\ln(x)}{72} - \frac{\ln(a)}{72} + \frac{-145a^4 x^4 + 360a^3 x^3 - 1080a^2 x^2 - 960ax + 1080}{4320a^4 x^4} \right. \\ \left. - \frac{(20a^3 x^3 - 20a^2 x^2 + 40ax + 360)e^{-ax}}{1440a^4 x^4} - \frac{(20a^3 x^3 + 480)(-\gamma - \ln(ax) - \text{Ei}_1(ax))}{1440a^3 x^3} \right)$$

Problem 15: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Ei}_3(ax)}{xa^2} dx$$

Optimal(type 4, 21 leaves, 1 step):

$$\frac{\text{Ei}_3(ax)}{2a^2} - \frac{\text{Ei}_1(ax)}{2a^2}$$

Result(type 4, 81 leaves):

$$\frac{\frac{\gamma}{2} + \frac{\ln(x)}{2} + \frac{\ln(a)}{2} - \frac{(-2 + \gamma + \ln(x) + \ln(a))x^2 a^2}{4} - \frac{a^2 x^2}{2} + \frac{(-9ax + 9)e^{-ax}}{36} + \frac{(-9a^2 x^2 + 18)(-\gamma - \ln(ax) - \text{Ei}_1(ax))}{36}}{a^2}$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Ei}_3(ax)}{a^2 x^5} dx$$

Optimal(type 4, 27 leaves, 1 step):

$$\frac{\text{Ei}_5(ax)}{2a^2 x^4} - \frac{\text{Ei}_3(ax)}{2a^2 x^4}$$

Result(type 4, 164 leaves):

$$a^2 \left( -\frac{1}{8a^4 x^4} + \frac{1}{3a^3 x^3} + \frac{-1 + \gamma + \ln(x) + \ln(a)}{4x^2 a^2} - \frac{1}{6ax} + \frac{31}{576} - \frac{\gamma}{48} - \frac{\ln(x)}{48} - \frac{\ln(a)}{48} + \frac{-155a^4 x^4 + 480a^3 x^3 + 720a^2 x^2 - 960ax + 360}{2880a^4 x^4} \right. \\ \left. - \frac{(15a^3 x^3 - 15a^2 x^2 - 150ax + 90)e^{-ax}}{720a^4 x^4} + \frac{(-15a^2 x^2 + 180)(-\gamma - \ln(ax) - \text{Ei}_1(ax))}{720a^2 x^2} \right)$$

Problem 19: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Ei}_4(ax)}{a^3 x^7} dx$$

Optimal(type 4, 27 leaves, 1 step):

$$\frac{\text{Ei}_7(ax)}{3a^3 x^6} - \frac{\text{Ei}_4(ax)}{3a^3 x^6}$$

Result(type 4, 212 leaves):

$$a^3 \left( -\frac{1}{18a^6 x^6} + \frac{1}{10x^5 a^5} - \frac{1}{8a^4 x^4} - \frac{-\frac{3}{2} + \gamma + \ln(x) + \ln(a)}{18x^3 a^3} + \frac{1}{48a^2 x^2} - \frac{1}{240ax} + \frac{167}{129600} - \frac{\gamma}{2160} - \frac{\ln(x)}{2160} - \frac{\ln(a)}{2160} \right. \\ \left. + \frac{-1169x^6 a^6 + 3780x^5 a^5 - 18900a^4 x^4 - 75600a^3 x^3 + 113400a^2 x^2 - 90720ax + 50400}{907200a^6 x^6} \right. \\ \left. - \frac{(28x^5 a^5 - 28a^4 x^4 + 56a^3 x^3 + 3192a^2 x^2 - 2688ax + 3360)e^{-ax}}{60480a^6 x^6} - \frac{(28a^3 x^3 + 3360)(-\gamma - \ln(ax) - \text{Ei}_1(ax))}{60480a^3 x^3} \right)$$

Problem 20: Unable to integrate problem.

$$\int \frac{\sqrt{\pi} \operatorname{erfc}(\sqrt{ax})}{x} dx$$

Optimal (type 5, 25 leaves, 1 step):

$$\ln(x) \sqrt{\pi} - 4 \operatorname{HypergeometricPFQ}\left(\left[\frac{1}{2}, \frac{1}{2}\right], \left[\frac{3}{2}, \frac{3}{2}\right], -ax\right) \sqrt{ax}$$

Result (type 8, 15 leaves):

$$\int \frac{\sqrt{\pi} \operatorname{erfc}(\sqrt{ax})}{x} dx$$

Problem 24: Result unnecessarily involves higher level functions.

$$\int \frac{(dx)^m \operatorname{Ei}_2(bx)}{bx} dx$$

Optimal (type 4, 55 leaves, 1 step):

$$\frac{(dx)^{1+m} \operatorname{Ei}_2(bx)}{bx d (1+m)} - \frac{(dx)^m \Gamma(m, bx)}{b (1+m) (bx)^m}$$

Result (type 5, 96 leaves):

$$b^{-1-m} (dx)^m x^{-m} \left( \frac{x^m b^m}{m} + \frac{(\Psi(1+m) + \gamma - 1 - \Psi(2+m) + \ln(x) + \ln(b)) x^{1+m} b^{1+m}}{1+m} - \frac{x^{2+m} b^{2+m} \operatorname{hypergeom}([1, 1, 2+m], [2, 3, 3+m], -bx)}{2(2+m)} \right)$$

Problem 25: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(dx)^m \operatorname{Ei}_3(bx)}{b^2 x^2} dx$$

Optimal (type 4, 57 leaves, 1 step):

$$\frac{(dx)^{1+m} \operatorname{Ei}_3(bx)}{b^2 x^2 d (1+m)} - \frac{(dx)^m \Gamma(-1+m, bx)}{b (1+m) (bx)^m}$$

Result (type 5, 115 leaves):

$$b^{-1-m} (dx)^m x^{-m} \left( \frac{x^{-1+m} b^{-1+m}}{2(-1+m)} - \frac{x^m b^m}{m} - \frac{(\Psi(1+m) + \gamma - \frac{3}{2} - \Psi(2+m) + \ln(x) + \ln(b)) x^{1+m} b^{1+m}}{2(1+m)} + \frac{x^{2+m} b^{2+m} \operatorname{hypergeom}([1, 1, 2+m], [2, 4, 3+m], -bx)}{6(2+m)} \right)$$

Problem 26: Unable to integrate problem.

$$\int \frac{\Gamma(n, ax)}{x^2} dx$$

Optimal(type 4, 20 leaves, 1 step):

$$a\Gamma(-1+n, ax) - \frac{\Gamma(n, ax)}{x}$$

Result(type 9, 147 leaves):

$$a \left( \frac{-\frac{\pi \csc(\pi n)}{\Gamma(1-n)xa} - \frac{\pi \csc(\pi n)}{\Gamma(2-n)}}{n(-1+n)(1+n)} x^{-1+n} a^{-1+n} (ax-n+1)(ax)^{-\frac{n}{2}} e^{-\frac{ax}{2}} \text{WhittakerM}\left(\frac{n}{2}, \frac{n}{2} + \frac{1}{2}, ax\right) \right. \\ \left. - \frac{x^{-2+n} a^{-2+n} (ax+1)(ax)^{-\frac{n}{2}} e^{-\frac{ax}{2}} \text{WhittakerM}\left(\frac{n}{2} + 1, \frac{n}{2} + \frac{1}{2}, ax\right)}{n(-1+n)} \right)$$

Problem 27: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\Gamma(n, ax)}{x^3} dx$$

Optimal(type 4, 23 leaves, 1 step):

$$\frac{a^2 \Gamma(-2+n, ax)}{2} - \frac{\Gamma(n, ax)}{2x^2}$$

Result(type 5, 78 leaves):

$$a^2 \left( -\frac{\pi \csc(\pi n)}{2\Gamma(1-n)x^2 a^2} + \frac{\pi \csc(\pi n)}{2\Gamma(3-n)} - \frac{x^{-2+n} a^{-2+n} \text{hypergeom}([n, -2+n], [1+n, -1+n], -ax)}{n(-2+n)} \right)$$

Problem 28: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\Gamma(n, 2x)}{x^3} dx$$

Optimal(type 4, 20 leaves, 1 step):

$$2\Gamma(-2+n, 2x) - \frac{\Gamma(n, 2x)}{2x^2}$$

Result(type 5, 68 leaves):

$$-\frac{\pi \csc(\pi n)}{2\Gamma(1-n)x^2} + \frac{2\pi \csc(\pi n)}{\Gamma(3-n)} - \frac{x^{-2+n} 2^n \text{hypergeom}([n, -2+n], [1+n, -1+n], -2x)}{n(-2+n)}$$



Problem 29: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\Gamma(n, 2x)}{x^4} dx$$

Optimal(type 4, 20 leaves, 1 step):

$$\frac{8\Gamma(-3+n, 2x)}{3} - \frac{\Gamma(n, 2x)}{3x^3}$$

Result(type 5, 68 leaves):

$$-\frac{\pi \csc(\pi n)}{3\Gamma(1-n)x^3} - \frac{8\pi \csc(\pi n)}{3\Gamma(4-n)} - \frac{x^{-3+n} 2^n \text{hypergeom}([n, -3+n], [1+n, -2+n], -2x)}{n(-3+n)}$$

Problem 30: Result more than twice size of optimal antiderivative.

$$\int (dx+c)^3 \text{Ei}_1(bx+a) dx$$

Optimal(type 4, 259 leaves, 8 steps):

$$\begin{aligned} & -\frac{(-ad+bc)^3 e^{-bx-a}}{4b^4} - \frac{(-ad+bc)^4 \text{Ei}_1(bx+a)}{4b^4 d} + \frac{(dx+c)^4 \text{Ei}_1(bx+a)}{4d} - \frac{d(-ad+bc)^2 e^{-a+\frac{bc}{d}} e^{-\frac{b(dx+c)}{d}} \left(1 + \frac{b(dx+c)}{d}\right)}{4b^4} \\ & - \frac{d^2(-ad+bc) e^{-a+\frac{bc}{d}} e^{-\frac{b(dx+c)}{d}} \left(1 + \frac{b(dx+c)}{d} + \frac{b^2(dx+c)^2}{2d^2}\right)}{2b^4} \\ & - \frac{3d^3 e^{-a+\frac{bc}{d}} e^{-\frac{b(dx+c)}{d}} \left(1 + \frac{b(dx+c)}{d} + \frac{b^2(dx+c)^2}{2d^2} + \frac{b^3(dx+c)^3}{6d^3}\right)}{2b^4} \end{aligned}$$

Result(type 4, 765 leaves):

$$\begin{aligned} & \frac{1}{b} \left( \frac{d^3 \text{Ei}_1(bx+a) (bx+a)^4}{4b^3} - \frac{d^3 \text{Ei}_1(bx+a) (bx+a)^3 a}{b^3} + \frac{d^2 \text{Ei}_1(bx+a) (bx+a)^3 c}{b^2} + \frac{3d^3 \text{Ei}_1(bx+a) (bx+a)^2 a^2}{2b^3} \right. \\ & - \frac{3d^2 \text{Ei}_1(bx+a) (bx+a)^2 a c}{b^2} + \frac{3d \text{Ei}_1(bx+a) (bx+a)^2 c^2}{2b} - \frac{d^3 \text{Ei}_1(bx+a) (bx+a) a^3}{b^3} + \frac{3d^2 \text{Ei}_1(bx+a) (bx+a) a^2 c}{b^2} \\ & - \frac{3d \text{Ei}_1(bx+a) (bx+a) a c^2}{b} + \text{Ei}_1(bx+a) (bx+a) c^3 + \frac{d^3 \text{Ei}_1(bx+a) a^4}{4b^3} - \frac{d^2 \text{Ei}_1(bx+a) a^3 c}{b^2} + \frac{3d \text{Ei}_1(bx+a) a^2 c^2}{2b} - \text{Ei}_1(bx+a) a c^3 \\ & + \frac{b \text{Ei}_1(bx+a) c^4}{4d} + \frac{1}{4b^3 d} \left( d^4 (-(bx+a)^3 e^{-bx-a} - 3(bx+a)^2 e^{-bx-a} - 6(bx+a) e^{-bx-a} - 6e^{-bx-a}) - a^4 d^4 \text{Ei}_1(bx+a) - b^4 c^4 \text{Ei}_1(bx \right. \\ & \left. + a) + 4e^{-bx-a} a^3 d^4 + 6a^2 d^4 (-(bx+a) e^{-bx-a} - e^{-bx-a}) + 4a d^4 ((bx+a)^2 e^{-bx-a} + 2(bx+a) e^{-bx-a} + 2e^{-bx-a}) - 4e^{-bx-a} b^3 c^3 d \right) \end{aligned}$$

$$+ 6b^2c^2d^2(-(bx+a)e^{-bx-a} - e^{-bx-a}) - 4bcd^3((bx+a)^2e^{-bx-a} + 2(bx+a)e^{-bx-a} + 2e^{-bx-a}) + 12e^{-bx-a}ab^2c^2d^2 - 12e^{-bx-a}a^2bcd^3 + 4ab^3c^3d\text{Ei}_1(bx+a) - 6a^2b^2c^2d^2\text{Ei}_1(bx+a) + 4a^3bcd^3\text{Ei}_1(bx+a) - 12abc d^3(-(bx+a)e^{-bx-a} - e^{-bx-a}))$$

Problem 37: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{-bx-a}(bx+a+1)}{(dx+c)^2} dx$$

Optimal(type 4, 79 leaves, 5 steps):

$$\frac{be^{-bx-a}}{d^2} - \frac{b(-ad+bc)e^{-a+\frac{bc}{d}}\text{Ei}_1\left(\frac{b(dx+c)}{d}\right)}{d^3} - \frac{e^{-bx-a}(bx+a+1)}{d(dx+c)}$$

Result(type 4, 209 leaves):

$$-\frac{1}{b} \left( \frac{b^2 \left( -\frac{e^{-bx-a}}{-bx-a+\frac{ad-bc}{d}} - e^{-\frac{ad-bc}{d}} \text{Ei}_1\left(bx+a-\frac{ad-bc}{d}\right) \right)}{d^2} \right. \\ \left. + \frac{(ad-bc)b^2 \left( -\frac{e^{-bx-a}}{-bx-a+\frac{ad-bc}{d}} - e^{-\frac{ad-bc}{d}} \text{Ei}_1\left(bx+a-\frac{ad-bc}{d}\right) \right)}{d^3} + \frac{b^2 e^{-\frac{ad-bc}{d}} \text{Ei}_1\left(bx+a-\frac{ad-bc}{d}\right)}{d^2} \right)$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{-bx-a}(bx+a+1)}{(dx+c)^4} dx$$

Optimal(type 4, 110 leaves, 5 steps):

$$-\frac{b(-ad+bc)e^{-a+\frac{bc}{d}}\text{Ei}_3\left(\frac{b(dx+c)}{d}\right)}{3(dx+c)^2d^3} + \frac{b^2e^{-a+\frac{bc}{d}}\text{Ei}_2\left(\frac{b(dx+c)}{d}\right)}{3(dx+c)d^3} - \frac{e^{-bx-a}(bx+a+1)}{3d(dx+c)^3}$$

Result(type 4, 411 leaves):

$$-\frac{1}{b} \left( \frac{b^4 \left( -\frac{e^{-bx-a}}{3\left(-bx-a+\frac{ad-bc}{d}\right)^3} - \frac{e^{-bx-a}}{6\left(-bx-a+\frac{ad-bc}{d}\right)^2} - \frac{e^{-bx-a}}{6\left(-bx-a+\frac{ad-bc}{d}\right)} - \frac{e^{-\frac{ad-bc}{d}}\text{Ei}_1\left(bx+a-\frac{ad-bc}{d}\right)}{6} \right)}{d^4} \right)$$

$$\begin{aligned}
& + \frac{1}{d^5} \left( b^4 (ad - bc) \left( -\frac{e^{-bx-a}}{3 \left( -bx-a + \frac{ad-bc}{d} \right)^3} - \frac{e^{-bx-a}}{6 \left( -bx-a + \frac{ad-bc}{d} \right)^2} - \frac{e^{-bx-a}}{6 \left( -bx-a + \frac{ad-bc}{d} \right)} \right. \right. \\
& \left. \left. - \frac{e^{-\frac{ad-bc}{d}} \operatorname{Ei}_1 \left( bx+a - \frac{ad-bc}{d} \right)}{6} \right) \right) \\
& - \frac{b^4 \left( -\frac{e^{-bx-a}}{2 \left( -bx-a + \frac{ad-bc}{d} \right)^2} - \frac{e^{-bx-a}}{2 \left( -bx-a + \frac{ad-bc}{d} \right)} - \frac{e^{-\frac{ad-bc}{d}} \operatorname{Ei}_1 \left( bx+a - \frac{ad-bc}{d} \right)}{2} \right)}{d^4}
\end{aligned}$$

Problem 40: Result more than twice size of optimal antiderivative.

$$\int \frac{2e^{-bx-a} \left( 1 + bx + a + \frac{(bx+a)^2}{2} \right)}{(dx+c)^2} dx$$

Optimal (type 4, 118 leaves, 6 steps):

$$-\frac{b(-ad+bc)e^{-bx-a}}{d^3} + \frac{b(-ad+bc)^2 e^{-a+\frac{bc}{d}} \operatorname{Ei}_1 \left( \frac{b(dx+c)}{d} \right)}{d^4} + \frac{be^{-bx-a}(bx+a+1)}{d^2} - \frac{2e^{-bx-a} \left( 1 + bx + a + \frac{(bx+a)^2}{2} \right)}{d(dx+c)}$$

Result (type 4, 376 leaves):

$$-\frac{1}{b} \left( \frac{b^2 e^{-bx-a}}{d^2} + \frac{(a^2 d^2 - 2abcd + c^2 b^2) b^2 \left( -\frac{e^{-bx-a}}{-bx-a + \frac{ad-bc}{d}} - e^{-\frac{ad-bc}{d}} \operatorname{Ei}_1 \left( bx+a - \frac{ad-bc}{d} \right) \right)}{d^4} \right)$$

$$\begin{aligned}
& + \frac{2(ad-bc)b^2 e^{-\frac{ad-bc}{d}} \operatorname{Ei}_1\left(bx+a-\frac{ad-bc}{d}\right)}{d^3} + \frac{2b^2 \left( -\frac{e^{-bx-a}}{-bx-a+\frac{ad-bc}{d}} - e^{-\frac{ad-bc}{d}} \operatorname{Ei}_1\left(bx+a-\frac{ad-bc}{d}\right) \right)}{d^2} \\
& + \frac{2(ad-bc)b^2 \left( -\frac{e^{-bx-a}}{-bx-a+\frac{ad-bc}{d}} - e^{-\frac{ad-bc}{d}} \operatorname{Ei}_1\left(bx+a-\frac{ad-bc}{d}\right) \right)}{d^3} + \frac{2b^2 e^{-\frac{ad-bc}{d}} \operatorname{Ei}_1\left(bx+a-\frac{ad-bc}{d}\right)}{d^2}
\end{aligned}$$

Problem 41: Unable to integrate problem.

$$\int \frac{\operatorname{Ei}_2(bx+a)}{(bx+a)(dx+c)^2} dx$$

Optimal(type 4, 111 leaves, 6 steps):

$$\frac{b \operatorname{Ei}_2(bx+a)}{(bx+a)d(-ad+bc)} - \frac{\operatorname{Ei}_2(bx+a)}{(bx+a)d(dx+c)} - \frac{b \operatorname{Ei}_1(bx+a)}{(-ad+bc)^2} + \frac{b e^{-a+\frac{bc}{d}} \operatorname{Ei}_1\left(\frac{b(dx+c)}{d}\right)}{(-ad+bc)^2}$$

Result(type 8, 24 leaves):

$$\int \frac{\operatorname{Ei}_2(bx+a)}{(bx+a)(dx+c)^2} dx$$

Problem 42: Unable to integrate problem.

$$\int \frac{(dx+c) \operatorname{Ei}_3(bx+a)}{(bx+a)^2} dx$$

Optimal(type 4, 99 leaves, 6 steps):

$$-\frac{(-ad+bc)^2 \operatorname{Ei}_3(bx+a)}{2(bx+a)^2 b^2 d} + \frac{(dx+c)^2 \operatorname{Ei}_3(bx+a)}{2(bx+a)^2 d} - \frac{(-ad+bc) \operatorname{Ei}_2(bx+a)}{(bx+a)b^2} - \frac{d \operatorname{Ei}_1(bx+a)}{2b^2}$$

Result(type 8, 22 leaves):

$$\int \frac{(dx+c) \operatorname{Ei}_3(bx+a)}{(bx+a)^2} dx$$

Problem 43: Unable to integrate problem.

$$\int \frac{\operatorname{Ei}_3(bx+a)}{(bx+a)^2} dx$$

Optimal(type 4, 38 leaves, 1 step):

$$\frac{\text{Ei}_3(bx+a)}{(bx+a)b} - \frac{\text{Ei}_2(bx+a)}{(bx+a)b}$$

Result(type 8, 17 leaves):

$$\int \frac{\text{Ei}_3(bx+a)}{(bx+a)^2} dx$$

Problem 44: Unable to integrate problem.

$$\int \frac{\text{Ei}_3(bx+a)}{(bx+a)^2(dx+c)^4} dx$$

Optimal(type 4, 240 leaves, 9 steps):

$$\begin{aligned} & \frac{b^3 \text{Ei}_3(bx+a)}{3(bx+a)^2 d(-ad+bc)^3} - \frac{\text{Ei}_3(bx+a)}{3(bx+a)^2 d(dx+c)^3} - \frac{b e^{-a+\frac{bc}{d}} d \text{Ei}_3\left(\frac{b(dx+c)}{d}\right)}{3(dx+c)^2(-ad+bc)^3} - \frac{b^3 \text{Ei}_2(bx+a)}{(bx+a)(-ad+bc)^4} - \frac{b^2 e^{-a+\frac{bc}{d}} d \text{Ei}_2\left(\frac{b(dx+c)}{d}\right)}{(dx+c)(-ad+bc)^4} \\ & + \frac{2b^3 d \text{Ei}_1(bx+a)}{(-ad+bc)^5} - \frac{2b^3 d e^{-a+\frac{bc}{d}} \text{Ei}_1\left(\frac{b(dx+c)}{d}\right)}{(-ad+bc)^5} \end{aligned}$$

Result(type 8, 24 leaves):

$$\int \frac{\text{Ei}_3(bx+a)}{(bx+a)^2(dx+c)^4} dx$$

Problem 45: Unable to integrate problem.

$$\int \frac{(dx+c) \text{Ei}_4(bx+a)}{(bx+a)^3} dx$$

Optimal(type 4, 106 leaves, 6 steps):

$$-\frac{(-ad+bc)^2 \text{Ei}_4(bx+a)}{2(bx+a)^3 b^2 d} + \frac{(dx+c)^2 \text{Ei}_4(bx+a)}{2(bx+a)^3 d} - \frac{(-ad+bc) \text{Ei}_3(bx+a)}{(bx+a)^2 b^2} - \frac{d \text{Ei}_2(bx+a)}{2(bx+a) b^2}$$

Result(type 8, 22 leaves):

$$\int \frac{(dx+c) \text{Ei}_4(bx+a)}{(bx+a)^3} dx$$

Problem 46: Unable to integrate problem.

$$\int \frac{\text{Ei}_4(bx+a)}{(bx+a)^3} dx$$

Optimal(type 4, 38 leaves, 1 step):

$$\frac{\text{Ei}_4(bx+a)}{(bx+a)^2 b} - \frac{\text{Ei}_3(bx+a)}{(bx+a)^2 b}$$

Result(type 8, 17 leaves):

$$\int \frac{\text{Ei}_4(bx+a)}{(bx+a)^3} dx$$

Problem 49: Unable to integrate problem.

$$\int (dx+c)^4 \Gamma(n, bx+a) dx$$

Optimal(type 4, 163 leaves, 9 steps):

$$\begin{aligned} & - \frac{(-ad+bc)^5 \Gamma(n, bx+a)}{5b^5 d} + \frac{(dx+c)^5 \Gamma(n, bx+a)}{5d} - \frac{(-ad+bc)^4 \Gamma(1+n, bx+a)}{b^5} - \frac{2d(-ad+bc)^3 \Gamma(2+n, bx+a)}{b^5} \\ & - \frac{2d^2(-ad+bc)^2 \Gamma(3+n, bx+a)}{b^5} - \frac{d^3(-ad+bc) \Gamma(4+n, bx+a)}{b^5} - \frac{d^4 \Gamma(5+n, bx+a)}{5b^5} \end{aligned}$$

Result(type 8, 17 leaves):

$$\int (dx+c)^4 \Gamma(n, bx+a) dx$$

Problem 50: Unable to integrate problem.

$$\int (dx+c)^2 \Gamma(n, bx+a) dx$$

Optimal(type 4, 109 leaves, 7 steps):

$$- \frac{(-ad+bc)^3 \Gamma(n, bx+a)}{3b^3 d} + \frac{(dx+c)^3 \Gamma(n, bx+a)}{3d} - \frac{(-ad+bc)^2 \Gamma(1+n, bx+a)}{b^3} - \frac{d(-ad+bc) \Gamma(2+n, bx+a)}{b^3} - \frac{d^2 \Gamma(3+n, bx+a)}{3b^3}$$

Result(type 8, 17 leaves):

$$\int (dx+c)^2 \Gamma(n, bx+a) dx$$

Problem 51: Unable to integrate problem.

$$\int x \Gamma(p, d(a+b \ln(cx^n))) dx$$

Optimal(type 4, 120 leaves, 4 steps):

$$\frac{x^2 \Gamma(p, d(a + b \ln(cx^n)))}{2} - \frac{x^2 \Gamma\left(p, -\frac{(-bdn+2)(a+b \ln(cx^n))}{bn}\right) (d(a+b \ln(cx^n)))^p}{2 e^{\frac{2a}{bn}} (cx^n)^{\frac{2}{n}} \left(-\frac{(-bdn+2)(a+b \ln(cx^n))}{bn}\right)^p}$$

Result(type 8, 18 leaves):

$$\int x \Gamma(p, d(a + b \ln(cx^n))) dx$$

Problem 52: Unable to integrate problem.

$$\int \frac{\Gamma(p, d(a + b \ln(cx^n)))}{x^2} dx$$

Optimal(type 4, 108 leaves, 4 steps):

$$-\frac{\Gamma(p, d(a + b \ln(cx^n)))}{x} + \frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} \Gamma\left(p, \frac{(bdn+1)(a+b \ln(cx^n))}{bn}\right) (d(a+b \ln(cx^n)))^p}{x \left(\frac{(bdn+1)(a+b \ln(cx^n))}{bn}\right)^p}$$

Result(type 8, 20 leaves):

$$\int \frac{\Gamma(p, d(a + b \ln(cx^n)))}{x^2} dx$$

Problem 53: Unable to integrate problem.

$$\int (dx + c) \ln\text{GAMMA}(bx + a) dx$$

Optimal(type 4, 30 leaves, 2 steps):

$$-\frac{d\Psi(-3, bx + a)}{b^2} + \frac{(dx + c) \Psi(-2, bx + a)}{b}$$

Result(type 8, 14 leaves):

$$\int (dx + c) \ln\text{GAMMA}(bx + a) dx$$

Problem 61: Unable to integrate problem.

$$\int \left( \frac{\Psi(1, bx + a)}{x^2} - \frac{b \Psi(2, bx + a)}{x} \right) dx$$

Optimal(type 4, 12 leaves, 2 steps):

$$-\frac{\Psi(1, bx + a)}{x}$$

Result(type 8, 27 leaves):

$$\int \left( \frac{\Psi(1, bx+a)}{x^2} - \frac{b\Psi(2, bx+a)}{x} \right) dx$$

Problem 62: Unable to integrate problem.

$$\int \left( \frac{\Psi(n, bx+a)}{x^2} - \frac{b\Psi(1+n, bx+a)}{x} \right) dx$$

Optimal(type 4, 12 leaves, 2 steps):

$$-\frac{\Psi(n, bx+a)}{x}$$

Result(type 8, 29 leaves):

$$\int \left( \frac{\Psi(n, bx+a)}{x^2} - \frac{b\Psi(1+n, bx+a)}{x} \right) dx$$

Test results for the 4 problems in "8.7 Zeta function.txt"

Problem 1: Unable to integrate problem.

$$\int x^2 \Psi(1, bx+a) dx$$

Optimal(type 4, 38 leaves, 4 steps):

$$-\frac{2x \ln \Gamma(bx+a)}{b^2} + \frac{2\Psi(-2, bx+a)}{b^3} + \frac{x^2 \Psi(bx+a)}{b}$$

Result(type 8, 13 leaves):

$$\int x^2 \Psi(1, bx+a) dx$$

Problem 3: Unable to integrate problem.

$$\int \left( \frac{\Psi(1, bx+a)}{x^2} - \frac{b\Psi(2, bx+a)}{x} \right) dx$$

Optimal(type 4, 12 leaves, 3 steps):

$$-\frac{\Psi(1, bx+a)}{x}$$

Result(type 8, 27 leaves):

$$\int \left( \frac{\Psi(1, bx+a)}{x^2} - \frac{b\Psi(2, bx+a)}{x} \right) dx$$



Problem 4: Unable to integrate problem.

$$\int \zeta(0, s, bx + a) dx$$

Optimal(type 4, 21 leaves, 1 step):

$$\frac{\zeta(0, -1 + s, bx + a)}{b(1 - s)}$$

Result(type 8, 10 leaves):

$$\int \zeta(0, s, bx + a) dx$$

Test results for the 51 problems in "8.8 Polylogarithm function.txt"

Problem 13: Result more than twice size of optimal antiderivative.

$$\int x^4 \text{polylog}(3, ax^2) dx$$

Optimal(type 4, 69 leaves, 6 steps):

$$\frac{8x}{125a^2} + \frac{8x^3}{375a} + \frac{8x^5}{625} - \frac{8 \operatorname{arctanh}(x\sqrt{a})}{125a^{5/2}} - \frac{4x^5 \ln(-ax^2 + 1)}{125} - \frac{2x^5 \text{polylog}(2, ax^2)}{25} + \frac{x^5 \text{polylog}(3, ax^2)}{5}$$

Result(type 4, 143 leaves):

$$-\frac{1}{2a^2\sqrt{-a}} \left( \frac{2x(-a)^{7/2}(168a^2x^4 + 280ax^2 + 840)}{13125a^3} + \frac{8x(-a)^{7/2}(\ln(1 - \sqrt{ax^2}) - \ln(1 + \sqrt{ax^2}))}{125a^3\sqrt{ax^2}} - \frac{8x^5(-a)^{7/2}\ln(-ax^2 + 1)}{125a} \right. \\ \left. - \frac{4x^5(-a)^{7/2}\text{polylog}(2, ax^2)}{25a} + \frac{2x^5(-a)^{7/2}\text{polylog}(3, ax^2)}{5a} \right)$$

Problem 14: Result more than twice size of optimal antiderivative.

$$\int \text{polylog}(3, ax^2) dx$$

Optimal(type 4, 46 leaves, 5 steps):

$$8x - 4x \ln(-ax^2 + 1) - 2x \text{polylog}(2, ax^2) + x \text{polylog}(3, ax^2) - \frac{8 \operatorname{arctanh}(x\sqrt{a})}{\sqrt{a}}$$

Result(type 4, 118 leaves):

$$-\frac{1}{2\sqrt{-a}} \left( \frac{16x(-a)^3/2}{a} + \frac{8x(-a)^3/2(\ln(1 - \sqrt{ax^2}) - \ln(1 + \sqrt{ax^2}))}{a\sqrt{ax^2}} - \frac{8x(-a)^3/2\ln(-ax^2 + 1)}{a} - \frac{4x(-a)^3/2\text{polylog}(2, ax^2)}{a} \right. \\ \left. + \frac{2x(-a)^3/2\text{polylog}(3, ax^2)}{a} \right)$$

Problem 29: Unable to integrate problem.

$$\int \left( \text{polylog} \left( -\frac{3}{2}, ax \right) + \text{polylog} \left( -\frac{1}{2}, ax \right) \right) dx$$

Optimal(type 4, 7 leaves, 2 steps):

$$x \text{polylog} \left( -\frac{1}{2}, ax \right)$$

Result(type 8, 13 leaves):

$$\int \left( \text{polylog} \left( -\frac{3}{2}, ax \right) + \text{polylog} \left( -\frac{1}{2}, ax \right) \right) dx$$

Problem 39: Unable to integrate problem.

$$\int x^2 \text{polylog}(3, c(bx+a)) dx$$

Optimal(type 4, 313 leaves, 33 steps):

$$\begin{aligned} & \frac{11a^2x}{18b^2} - \frac{5a(-ca+1)x}{36cb^2} + \frac{(-ca+1)^2x}{27c^2b^2} - \frac{5ax^2}{72b} + \frac{(-ca+1)x^2}{54bc} + \frac{x^3}{81} - \frac{5a(-ca+1)^2 \ln(-bcx-ca+1)}{36b^3c^2} + \frac{(-ca+1)^3 \ln(-bcx-ca+1)}{27b^3c^3} \\ & + \frac{5ax^2 \ln(-bcx-ca+1)}{36b} - \frac{x^3 \ln(-bcx-ca+1)}{27} + \frac{11a^2(-bcx-ca+1) \ln(-bcx-ca+1)}{18b^3c} - \frac{11a^3 \text{polylog}(2, c(bx+a))}{18b^3} \\ & - \frac{a^2x \text{polylog}(2, c(bx+a))}{3b^2} + \frac{ax^2 \text{polylog}(2, c(bx+a))}{6b} - \frac{x^3 \text{polylog}(2, c(bx+a))}{9} + \frac{2a^3 \text{polylog}(3, c(bx+a))}{3b^3} \\ & - \frac{(-b^3x^3 + a^3) \text{polylog}(3, c(bx+a))}{3b^3} \end{aligned}$$

Result(type 8, 15 leaves):

$$\int x^2 \text{polylog}(3, c(bx+a)) dx$$

Problem 40: Unable to integrate problem.

$$\int \text{polylog}(3, c(bx+a)) dx$$

Optimal(type 4, 84 leaves, 9 steps):

$$x + \frac{(-bcx-ca+1) \ln(-bcx-ca+1)}{bc} - \frac{a \text{polylog}(2, c(bx+a))}{b} - x \text{polylog}(2, c(bx+a)) + \frac{a \text{polylog}(3, c(bx+a))}{b} + x \text{polylog}(3, c(bx+a))$$

Result(type 8, 11 leaves):

$$\int \text{polylog}(3, c(bx+a)) dx$$

Problem 42: Unable to integrate problem.

$$\int \frac{\text{polylog}(2, x)}{(-1+x)x} dx$$

Optimal(type 4, 51 leaves, 8 steps):

$$\ln(1-x)^2 \ln(x) + 2 \ln(1-x) \text{polylog}(2, 1-x) + \ln(1-x) \text{polylog}(2, x) - 2 \text{polylog}(3, 1-x) - \text{polylog}(3, x)$$

Result(type 8, 14 leaves):

$$\int \frac{\text{polylog}(2, x)}{(-1+x)x} dx$$

Problem 43: Unable to integrate problem.

$$\int -\frac{\text{polylog}(2, x)}{(1-x)x} dx$$

Optimal(type 4, 51 leaves, 8 steps):

$$\ln(1-x)^2 \ln(x) + 2 \ln(1-x) \text{polylog}(2, 1-x) + \ln(1-x) \text{polylog}(2, x) - 2 \text{polylog}(3, 1-x) - \text{polylog}(3, x)$$

Result(type 8, 17 leaves):

$$\int -\frac{\text{polylog}(2, x)}{(1-x)x} dx$$

Problem 44: Unable to integrate problem.

$$\int -\frac{\ln\left(1 - e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(bx+a)(dx+c)} dx$$

Optimal(type 4, 33 leaves, 1 step):

$$\frac{\text{polylog}\left(2, e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(-ad+bc)n}$$

Result(type 8, 39 leaves):

$$\int -\frac{\ln\left(1 - e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(bx+a)(dx+c)} dx$$

Problem 46: Unable to integrate problem.

$$\int x^3 \text{polylog}(n, d(F^{c(bx+a)})^p) dx$$

Optimal(type 4, 135 leaves, 5 steps):

$$\frac{x^3 \text{polylog}(1+n, d(F^{c(bx+a)})^p)}{bc p \ln(F)} - \frac{3x^2 \text{polylog}(2+n, d(F^{c(bx+a)})^p)}{b^2 c^2 p^2 \ln(F)^2} + \frac{6x \text{polylog}(3+n, d(F^{c(bx+a)})^p)}{b^3 c^3 p^3 \ln(F)^3} - \frac{6 \text{polylog}(4+n, d(F^{c(bx+a)})^p)}{b^4 c^4 p^4 \ln(F)^4}$$

Result(type 8, 21 leaves):

$$\int x^3 \operatorname{polylog}(n, d (F^c(bx+a))^p) dx$$

Problem 48: Unable to integrate problem.

$$\int \frac{\ln(-cx+1) \operatorname{polylog}(2, cx)}{x^3} dx$$

Optimal(type 4, 175 leaves, 23 steps):

$$\begin{aligned} & -c^2 \ln(x) + c^2 \ln(-cx+1) - \frac{c \ln(-cx+1)}{x} - \frac{c^2 \ln(-cx+1)^2}{4} + \frac{\ln(-cx+1)^2}{4x^2} + \frac{c^2 \ln(cx) \ln(-cx+1)^2}{2} - \frac{c^2 \operatorname{polylog}(2, cx)}{2} + \frac{c \operatorname{polylog}(2, cx)}{2x} \\ & + \frac{c^2 \ln(-cx+1) \operatorname{polylog}(2, cx)}{2} - \frac{\ln(-cx+1) \operatorname{polylog}(2, cx)}{2x^2} + c^2 \ln(-cx+1) \operatorname{polylog}(2, -cx+1) - \frac{c^2 \operatorname{polylog}(3, cx)}{2} - c^2 \operatorname{polylog}(3, -cx+1) \end{aligned}$$

Result(type 8, 18 leaves):

$$\int \frac{\ln(-cx+1) \operatorname{polylog}(2, cx)}{x^3} dx$$

Problem 49: Unable to integrate problem.

$$\int \frac{(bx+a) \ln(-cx+1) \operatorname{polylog}(2, cx)}{x^2} dx$$

Optimal(type 4, 129 leaves, 13 steps):

$$\begin{aligned} & \frac{a(-cx+1) \ln(-cx+1)^2}{x} + ac \ln(cx) \ln(-cx+1)^2 - 2ac \operatorname{polylog}(2, cx) + ac \ln(-cx+1) \operatorname{polylog}(2, cx) - \frac{a \ln(-cx+1) \operatorname{polylog}(2, cx)}{x} \\ & - \frac{b \operatorname{polylog}(2, cx)^2}{2} + 2ac \ln(-cx+1) \operatorname{polylog}(2, -cx+1) - ac \operatorname{polylog}(3, cx) - 2ac \operatorname{polylog}(3, -cx+1) \end{aligned}$$

Result(type 8, 23 leaves):

$$\int \frac{(bx+a) \ln(-cx+1) \operatorname{polylog}(2, cx)}{x^2} dx$$

Problem 50: Unable to integrate problem.

$$\int \frac{(bx+a) \ln(-cx+1) \operatorname{polylog}(2, cx)}{x^4} dx$$

Optimal(type 4, 410 leaves, 41 steps):

$$\begin{aligned} & -\frac{7ac \ln(-cx+1)}{36x^2} - \frac{bc \ln(-cx+1)}{2x} - \frac{2ac^2 \ln(-cx+1)}{9x} - \frac{c(2ca+3b) \ln(-cx+1)}{6x} + \frac{c^2(2ca+3b) \ln(cx) \ln(-cx+1)^2}{6} \\ & + \frac{c^2(2ca+3b) \ln(-cx+1) \operatorname{polylog}(2, cx)}{6} + \frac{c^2(2ca+3b) \ln(-cx+1) \operatorname{polylog}(2, -cx+1)}{3} + \frac{7ac^2}{36x} - \frac{bc^2 \operatorname{polylog}(2, cx)}{2} \end{aligned}$$

$$\begin{aligned}
& - \frac{2ac^3 \operatorname{polylog}(2, cx)}{9} - \frac{c^2(2ca+3b) \operatorname{polylog}(3, cx)}{6} - \frac{c^2(2ca+3b) \operatorname{polylog}(3, -cx+1)}{3} - \frac{c^2(2ca+3b) \ln(x)}{6} - \frac{bc^2 \ln(x)}{2} - \frac{5ac^3 \ln(x)}{12} \\
& + \frac{bc^2 \ln(-cx+1)}{2} + \frac{5ac^3 \ln(-cx+1)}{12} + \frac{c^2(2ca+3b) \ln(-cx+1)}{6} - \frac{bc^2 \ln(-cx+1)^2}{4} - \frac{ac^3 \ln(-cx+1)^2}{9} + \frac{a \ln(-cx+1)^2}{9x^3} \\
& + \frac{b \ln(-cx+1)^2}{4x^2} - \frac{\left(\frac{2a}{x^3} + \frac{3b}{x^2}\right) \ln(-cx+1) \operatorname{polylog}(2, cx)}{6} + \frac{ac \operatorname{polylog}(2, cx)}{6x^2} + \frac{c(2ca+3b) \operatorname{polylog}(2, cx)}{6x}
\end{aligned}$$

Result(type 8, 23 leaves):

$$\int \frac{(bx+a) \ln(-cx+1) \operatorname{polylog}(2, cx)}{x^4} dx$$

Problem 51: Unable to integrate problem.

$$\int \frac{(cx^2+bx+a) \ln(-dx+1) \operatorname{polylog}(2, dx)}{x^4} dx$$

Optimal(type 4, 465 leaves, 43 steps):

$$\begin{aligned}
& \frac{d(6c+d(2ad+3b)) \ln(-dx+1) \operatorname{polylog}(2, dx)}{6} + \frac{d(6c+d(2ad+3b)) \ln(-dx+1) \operatorname{polylog}(2, -dx+1)}{3} + \frac{c(-dx+1) \ln(-dx+1)^2}{x} \\
& - \frac{7ad \ln(-dx+1)}{36x^2} - \frac{bd \ln(-dx+1)}{2x} + \frac{ad \operatorname{polylog}(2, dx)}{6x^2} + \frac{d(2ad+3b) \operatorname{polylog}(2, dx)}{6x} - \frac{2ad^2 \ln(-dx+1)}{9x} - \frac{d(2ad+3b) \ln(-dx+1)}{6x} \\
& + \frac{d(6c+d(2ad+3b)) \ln(dx) \ln(-dx+1)^2}{6} + \frac{7ad^2}{36x} - 2cd \operatorname{polylog}(2, dx) - \frac{bd^2 \operatorname{polylog}(2, dx)}{2} - \frac{2ad^3 \operatorname{polylog}(2, dx)}{9} \\
& - \frac{d(6c+d(2ad+3b)) \operatorname{polylog}(3, dx)}{6} - \frac{d(6c+d(2ad+3b)) \operatorname{polylog}(3, -dx+1)}{3} - \frac{bd^2 \ln(x)}{2} - \frac{5ad^3 \ln(x)}{12} - \frac{d^2(2ad+3b) \ln(x)}{6} \\
& + \frac{bd^2 \ln(-dx+1)}{2} + \frac{5ad^3 \ln(-dx+1)}{12} + \frac{d^2(2ad+3b) \ln(-dx+1)}{6} - \frac{bd^2 \ln(-dx+1)^2}{4} - \frac{ad^3 \ln(-dx+1)^2}{9} + \frac{a \ln(-dx+1)^2}{9x^3} \\
& + \frac{b \ln(-dx+1)^2}{4x^2} - \frac{\left(\frac{2a}{x^3} + \frac{3b}{x^2} + \frac{6c}{x}\right) \ln(-dx+1) \operatorname{polylog}(2, dx)}{6}
\end{aligned}$$

Result(type 8, 28 leaves):

$$\int \frac{(cx^2+bx+a) \ln(-dx+1) \operatorname{polylog}(2, dx)}{x^4} dx$$

Test results for the 107 problems in "8.9 Product logarithm function.txt"

Problem 5: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(c \operatorname{LambertW}(bx+a))^{5/2}} dx$$

Optimal(type 4, 70 leaves, 3 steps):

$$-\frac{2(bx+a)}{3b(c \operatorname{LambertW}(bx+a))^{5/2}} - \frac{10(bx+a)}{3bc(c \operatorname{LambertW}(bx+a))^{3/2}} + \frac{10 \operatorname{erfi}\left(\frac{\sqrt{c \operatorname{LambertW}(bx+a)}}{\sqrt{c}}\right) \sqrt{\pi}}{3bc^5/2}$$

Result(type 4, 159 leaves):

$$\frac{1}{bc^2} \left( 2 \left( -\frac{bx+a}{\sqrt{c \operatorname{LambertW}(bx+a)} \operatorname{LambertW}(bx+a)} + \frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{-\frac{1}{c}} \sqrt{c \operatorname{LambertW}(bx+a)}\right)}{c \sqrt{-\frac{1}{c}}} \right) + c \left( -\frac{bx+a}{3(c \operatorname{LambertW}(bx+a))^{3/2} \operatorname{LambertW}(bx+a)} + \frac{2 \left( -\frac{bx+a}{\sqrt{c \operatorname{LambertW}(bx+a)} \operatorname{LambertW}(bx+a)} + \frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{-\frac{1}{c}} \sqrt{c \operatorname{LambertW}(bx+a)}\right)}{c \sqrt{-\frac{1}{c}}} \right)}{3c} \right) \right)$$

Problem 6: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(c \operatorname{LambertW}(bx+a))^{7/2}} dx$$

Optimal(type 4, 93 leaves, 4 steps):

$$-\frac{2(bx+a)}{5b(c \operatorname{LambertW}(bx+a))^{7/2}} - \frac{14(bx+a)}{15bc(c \operatorname{LambertW}(bx+a))^{5/2}} - \frac{28(bx+a)}{15bc^2(c \operatorname{LambertW}(bx+a))^{3/2}} + \frac{28 \operatorname{erfi}\left(\frac{\sqrt{c \operatorname{LambertW}(bx+a)}}{\sqrt{c}}\right) \sqrt{\pi}}{15bc^7/2}$$

Result(type 4, 221 leaves):

$$\begin{aligned}
& \frac{1}{bc^2} \left( 2 \left( -\frac{bx+a}{3(c \operatorname{LambertW}(bx+a))^{3/2} \operatorname{LambertW}(bx+a)} \right. \right. \\
& \quad \left. \left. + \frac{2 \left( -\frac{bx+a}{\sqrt{c \operatorname{LambertW}(bx+a)} \operatorname{LambertW}(bx+a)} + \frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{-\frac{1}{c}} \sqrt{c \operatorname{LambertW}(bx+a)}\right)}{c \sqrt{-\frac{1}{c}}} \right)}{3c} \right) + c \right) \\
& - \frac{bx+a}{5(c \operatorname{LambertW}(bx+a))^{5/2} \operatorname{LambertW}(bx+a)} + \frac{1}{5c} \left( 2 \left( -\frac{bx+a}{3(c \operatorname{LambertW}(bx+a))^{3/2} \operatorname{LambertW}(bx+a)} \right. \right. \\
& \quad \left. \left. + \frac{2 \left( -\frac{bx+a}{\sqrt{c \operatorname{LambertW}(bx+a)} \operatorname{LambertW}(bx+a)} + \frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{-\frac{1}{c}} \sqrt{c \operatorname{LambertW}(bx+a)}\right)}{c \sqrt{-\frac{1}{c}}} \right)}{3c} \right) \right) \right)
\end{aligned}$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(-c \operatorname{LambertW}(bx+a))^{7/2}} dx$$

Optimal(type 4, 97 leaves, 4 steps):

$$-\frac{2(bx+a)}{5b(-c \operatorname{LambertW}(bx+a))^{7/2}} + \frac{14(bx+a)}{15bc(-c \operatorname{LambertW}(bx+a))^{5/2}} - \frac{28(bx+a)}{15bc^2(-c \operatorname{LambertW}(bx+a))^{3/2}} + \frac{28 \operatorname{erf}\left(\frac{\sqrt{-c \operatorname{LambertW}(bx+a)}}{\sqrt{c}}\right) \sqrt{\pi}}{15bc^{7/2}}$$

Result(type 4, 209 leaves):

$$\frac{1}{bc^2} \left( 2 \left( -\frac{bx+a}{3(-c \operatorname{LambertW}(bx+a))^3/2 \operatorname{LambertW}(bx+a)} \right. \right. \\ \left. \left. - \frac{2 \left( -\frac{bx+a}{\sqrt{-c \operatorname{LambertW}(bx+a)} \operatorname{LambertW}(bx+a)} - \frac{\sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{-c \operatorname{LambertW}(bx+a)}}{\sqrt{c}}\right)}{\sqrt{c}} \right)}{3c} \right) - c \right) \\ - \frac{bx+a}{5(-c \operatorname{LambertW}(bx+a))^5/2 \operatorname{LambertW}(bx+a)} \\ - \frac{1}{5c} \left( 2 \left( -\frac{bx+a}{3(-c \operatorname{LambertW}(bx+a))^3/2 \operatorname{LambertW}(bx+a)} \right. \right. \\ \left. \left. - \frac{2 \left( -\frac{bx+a}{\sqrt{-c \operatorname{LambertW}(bx+a)} \operatorname{LambertW}(bx+a)} - \frac{\sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{-c \operatorname{LambertW}(bx+a)}}{\sqrt{c}}\right)}{\sqrt{c}} \right)}{3c} \right) \right) \right)$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c \operatorname{LambertW}(ax)}}{x^3} dx$$

Optimal (type 4, 60 leaves, 3 steps):

$$\frac{2(c \operatorname{LambertW}(ax))^3/2}{3cx^2} + \frac{2a^2 \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{c \operatorname{LambertW}(ax)}}{\sqrt{c}}\right) \sqrt{c} \sqrt{2} \sqrt{\pi}}{3} - \frac{2\sqrt{c \operatorname{LambertW}(ax)}}{3x^2}$$

Result (type 4, 121 leaves):



$$2 a^2 c \left( -\frac{e^{-2 \operatorname{LambertW}(a x)}}{\sqrt{c \operatorname{LambertW}(a x)}} - \frac{\sqrt{\pi} \sqrt{2} \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{c \operatorname{LambertW}(a x)}}{\sqrt{c}}\right)}{\sqrt{c}} \right) + c \left( -\frac{e^{-2 \operatorname{LambertW}(a x)}}{3 (c \operatorname{LambertW}(a x))^{3/2}} \right. \\ \left. - \frac{4 \left( -\frac{e^{-2 \operatorname{LambertW}(a x)}}{\sqrt{c \operatorname{LambertW}(a x)}} - \frac{\sqrt{\pi} \sqrt{2} \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{c \operatorname{LambertW}(a x)}}{\sqrt{c}}\right)}{\sqrt{c}} \right)}{3 c} \right) \right)$$

Problem 42: Unable to integrate problem.

$$\int \frac{x^m}{\operatorname{LambertW}(a x)} dx$$

Optimal(type 4, 101 leaves, 3 steps):

$$\frac{x^m \Gamma(m, -(1+m) \operatorname{LambertW}(a x)) (-(1+m) \operatorname{LambertW}(a x))^{1-m}}{a e^m \operatorname{LambertW}(a x) (1+m) \operatorname{LambertW}(a x)} + \frac{x^m \Gamma(1+m, -(1+m) \operatorname{LambertW}(a x))}{a e^m \operatorname{LambertW}(a x) (1+m) (-(1+m) \operatorname{LambertW}(a x))^m}$$

Result(type 8, 12 leaves):

$$\int \frac{x^m}{\operatorname{LambertW}(a x)} dx$$

Problem 43: Unable to integrate problem.

$$\int \frac{x^m}{\operatorname{LambertW}(a x)^2} dx$$

Optimal(type 4, 109 leaves, 3 steps):

$$\frac{x^m \Gamma(m, -(1+m) \operatorname{LambertW}(a x)) (-(1+m) \operatorname{LambertW}(a x))^{1-m}}{a e^m \operatorname{LambertW}(a x) (1+m) \operatorname{LambertW}(a x)} + \frac{x^m \Gamma(-1+m, -(1+m) \operatorname{LambertW}(a x)) (-(1+m) \operatorname{LambertW}(a x))^{2-m}}{a e^m \operatorname{LambertW}(a x) (1+m) \operatorname{LambertW}(a x)^2}$$

Result(type 8, 12 leaves):

$$\int \frac{x^m}{\operatorname{LambertW}(a x)^2} dx$$

Problem 49: Unable to integrate problem.

$$\int \frac{\operatorname{LambertW}(a x^2)^3}{x^9} dx$$

Optimal(type 4, 28 leaves, 2 steps):

$$-\frac{3a^4 \operatorname{Ei}(-4 \operatorname{LambertW}(ax^2))}{2} - \frac{\operatorname{LambertW}(ax^2)^3}{2x^8}$$

Result(type 8, 14 leaves):

$$\int \frac{\operatorname{LambertW}(ax^2)^3}{x^9} dx$$

Problem 51: Unable to integrate problem.

$$\int \frac{1}{x^3 \operatorname{LambertW}(ax^2)} dx$$

Optimal(type 4, 31 leaves, 4 steps):

$$-\frac{1}{4x^2} - \frac{a \operatorname{Ei}(-\operatorname{LambertW}(ax^2))}{4} - \frac{1}{4x^2 \operatorname{LambertW}(ax^2)}$$

Result(type 8, 14 leaves):

$$\int \frac{1}{x^3 \operatorname{LambertW}(ax^2)} dx$$

Problem 52: Unable to integrate problem.

$$\int \frac{x^7}{\operatorname{LambertW}(ax^2)^2} dx$$

Optimal(type 4, 40 leaves, 3 steps):

$$-\frac{x^8}{64 \operatorname{LambertW}(ax^2)^4} + \frac{x^8}{16 \operatorname{LambertW}(ax^2)^3} + \frac{x^8}{8 \operatorname{LambertW}(ax^2)^2}$$

Result(type 8, 14 leaves):

$$\int \frac{x^7}{\operatorname{LambertW}(ax^2)^2} dx$$

Problem 58: Unable to integrate problem.

$$\int \frac{\sqrt{c \operatorname{LambertW}(ax^2)}}{x^3} dx$$

Optimal(type 4, 40 leaves, 2 steps):

$$-\frac{a \operatorname{erf}\left(\frac{\sqrt{c \operatorname{LambertW}(ax^2)}}{\sqrt{c}}\right) \sqrt{c} \sqrt{\pi}}{2} - \frac{\sqrt{c \operatorname{LambertW}(ax^2)}}{x^2}$$

Result(type 8, 16 leaves):

$$\int \frac{\sqrt{c \operatorname{LambertW}(ax^2)}}{x^3} dx$$

Problem 59: Unable to integrate problem.

$$\int \frac{\sqrt{c \operatorname{LambertW}(ax^2)}}{x^7} dx$$

Optimal(type 4, 84 leaves, 4 steps):

$$\frac{(c \operatorname{LambertW}(ax^2))^{3/2}}{15 cx^6} - \frac{2 (c \operatorname{LambertW}(ax^2))^{5/2}}{5 c^2 x^6} - \frac{2 a^3 \operatorname{erf}\left(\frac{\sqrt{3} \sqrt{c \operatorname{LambertW}(ax^2)}}{\sqrt{c}}\right) \sqrt{c} \sqrt{3} \sqrt{\pi}}{5} - \frac{\sqrt{c \operatorname{LambertW}(ax^2)}}{5 x^6}$$

Result(type 8, 16 leaves):

$$\int \frac{\sqrt{c \operatorname{LambertW}(ax^2)}}{x^7} dx$$

Problem 60: Unable to integrate problem.

$$\int \frac{x^5}{\sqrt{c \operatorname{LambertW}(ax^2)}} dx$$

Optimal(type 4, 82 leaves, 4 steps):

$$-\frac{c^2 x^6}{72 (c \operatorname{LambertW}(ax^2))^{5/2}} + \frac{cx^6}{36 (c \operatorname{LambertW}(ax^2))^{3/2}} + \frac{\operatorname{erfi}\left(\frac{\sqrt{3} \sqrt{c \operatorname{LambertW}(ax^2)}}{\sqrt{c}}\right) \sqrt{3} \sqrt{\pi}}{432 a^3 \sqrt{c}} + \frac{x^6}{6 \sqrt{c \operatorname{LambertW}(ax^2)}}$$

Result(type 8, 16 leaves):

$$\int \frac{x^5}{\sqrt{c \operatorname{LambertW}(ax^2)}} dx$$

Problem 61: Unable to integrate problem.

$$\int \frac{x^3}{\sqrt{c \operatorname{LambertW}(ax^2)}} dx$$

Optimal(type 4, 64 leaves, 3 steps):

$$\frac{cx^4}{16 (c \operatorname{LambertW}(ax^2))^3 / 2} - \frac{\operatorname{erfi}\left(\frac{\sqrt{2} \sqrt{c \operatorname{LambertW}(ax^2)}}{\sqrt{c}}\right) \sqrt{2} \sqrt{\pi}}{64 a^2 \sqrt{c}} + \frac{x^4}{4 \sqrt{c \operatorname{LambertW}(ax^2)}}$$

Result(type 8, 16 leaves):

$$\int \frac{x^3}{\sqrt{c \operatorname{LambertW}(ax^2)}} dx$$

Problem 62: Unable to integrate problem.

$$\int \frac{x^2}{\sqrt{c \operatorname{LambertW}(ax^2)}} dx$$

Optimal(type 4, 32 leaves, 2 steps):

$$\frac{cx^3}{9 (c \operatorname{LambertW}(ax^2))^3 / 2} + \frac{x^3}{3 \sqrt{c \operatorname{LambertW}(ax^2)}}$$

Result(type 8, 16 leaves):

$$\int \frac{x^2}{\sqrt{c \operatorname{LambertW}(ax^2)}} dx$$

Problem 64: Unable to integrate problem.

$$\int \frac{1}{x^5 \sqrt{c \operatorname{LambertW}(ax^2)}} dx$$

Optimal(type 4, 84 leaves, 4 steps):

$$\frac{4 (c \operatorname{LambertW}(ax^2))^3 / 2}{15 c^2 x^4} + \frac{4 a^2 \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{c \operatorname{LambertW}(ax^2)}}{\sqrt{c}}\right) \sqrt{2} \sqrt{\pi}}{15 \sqrt{c}} - \frac{1}{5 x^4 \sqrt{c \operatorname{LambertW}(ax^2)}} - \frac{\sqrt{c \operatorname{LambertW}(ax^2)}}{15 c x^4}$$

Result(type 8, 16 leaves):

$$\int \frac{1}{x^5 \sqrt{c \operatorname{LambertW}(ax^2)}} dx$$

Problem 66: Result more than twice size of optimal antiderivative.

$$\int x (c \operatorname{LambertW}(ax^2))^p dx$$

Optimal(type 4, 56 leaves, 3 steps):

$$\frac{x^2 (c \operatorname{LambertW}(ax^2))^p}{2} - \frac{p \Gamma(1+p, -\operatorname{LambertW}(ax^2)) (c \operatorname{LambertW}(ax^2))^p}{2a (-\operatorname{LambertW}(ax^2))^p}$$

Result(type 4, 253 leaves):

$$\begin{aligned} & \frac{1}{2a} \left( (-1)^{-p} (c \operatorname{LambertW}(ax^2))^p \operatorname{LambertW}(ax^2)^{-p} \left( \operatorname{LambertW}(ax^2)^p (-1)^p (1+p) p \Gamma(p) (-\operatorname{LambertW}(ax^2))^{-p} \right. \right. \\ & \quad \left. \left. + \frac{\operatorname{LambertW}(ax^2)^p (-1)^p (\operatorname{LambertW}(ax^2) - p - 1) ax^2}{\operatorname{LambertW}(ax^2)} - \operatorname{LambertW}(ax^2)^p (-1)^p (1+p) p (-\operatorname{LambertW}(ax^2))^{-p} \Gamma(p, -\operatorname{LambertW}(ax^2)) \right) \right) \\ & - (-1)^{-p} (c \operatorname{LambertW}(ax^2))^p \operatorname{LambertW}(ax^2)^{-p} \left( \operatorname{LambertW}(ax^2)^p (-1)^p p \Gamma(p) (-\operatorname{LambertW}(ax^2))^{-p} - \frac{\operatorname{LambertW}(ax^2)^p (-1)^p ax^2}{\operatorname{LambertW}(ax^2)} \right. \\ & \quad \left. - \operatorname{LambertW}(ax^2)^p (-1)^p p (-\operatorname{LambertW}(ax^2))^{-p} \Gamma(p, -\operatorname{LambertW}(ax^2)) \right) \end{aligned}$$

Problem 67: Unable to integrate problem.

$$\int \frac{(c \operatorname{LambertW}(ax^2))^p}{x} dx$$

Optimal(type 4, 38 leaves, 2 steps):

$$\frac{(c \operatorname{LambertW}(ax^2))^p}{2p} + \frac{(c \operatorname{LambertW}(ax^2))^{1+p}}{2c(1+p)}$$

Result(type 8, 16 leaves):

$$\int \frac{(c \operatorname{LambertW}(ax^2))^p}{x} dx$$

Problem 73: Unable to integrate problem.

$$\int x^2 \sqrt{\operatorname{LambertW}\left(\frac{a}{x}\right)} dx$$

Optimal(type 4, 64 leaves, 4 steps):

$$-\frac{2x^3 \operatorname{LambertW}\left(\frac{a}{x}\right)^{3/2}}{15} + \frac{4x^3 \operatorname{LambertW}\left(\frac{a}{x}\right)^{5/2}}{5} + \frac{4a^3 \operatorname{erf}\left(\sqrt{3} \sqrt{\operatorname{LambertW}\left(\frac{a}{x}\right)}\right) \sqrt{3} \sqrt{\pi}}{5} + \frac{2x^3 \sqrt{\operatorname{LambertW}\left(\frac{a}{x}\right)}}{5}$$

Result(type 8, 14 leaves):

$$\int x^2 \sqrt{\operatorname{LambertW}\left(\frac{a}{x}\right)} dx$$

Problem 75: Unable to integrate problem.

$$\int \frac{1}{x^4 \sqrt{\text{LambertW}\left(\frac{a}{x}\right)}} dx$$

Optimal(type 4, 64 leaves, 4 steps):

$$\frac{1}{36 x^3 \text{LambertW}\left(\frac{a}{x}\right)^{5/2}} - \frac{1}{18 x^3 \text{LambertW}\left(\frac{a}{x}\right)^{3/2}} - \frac{\text{erfi}\left(\sqrt{3} \sqrt{\text{LambertW}\left(\frac{a}{x}\right)}\right) \sqrt{3} \sqrt{\pi}}{216 a^3} - \frac{1}{3 x^3 \sqrt{\text{LambertW}\left(\frac{a}{x}\right)}}$$

Result(type 8, 14 leaves):

$$\int \frac{1}{x^4 \sqrt{\text{LambertW}\left(\frac{a}{x}\right)}} dx$$

Problem 76: Unable to integrate problem.

$$\int x^2 \left( c \text{LambertW}\left(\frac{a}{x}\right) \right)^p dx$$

Optimal(type 4, 120 leaves, 4 steps):

$$\frac{3^{3-p} e^{4 \text{LambertW}\left(\frac{a}{x}\right)} x^4 \Gamma\left(-3+p, 3 \text{LambertW}\left(\frac{a}{x}\right)\right) \text{LambertW}\left(\frac{a}{x}\right)^{4-p} \left( c \text{LambertW}\left(\frac{a}{x}\right) \right)^p}{a} + \frac{3^{2-p} e^{4 \text{LambertW}\left(\frac{a}{x}\right)} x^4 \Gamma\left(-2+p, 3 \text{LambertW}\left(\frac{a}{x}\right)\right) \text{LambertW}\left(\frac{a}{x}\right)^{3-p} \left( c \text{LambertW}\left(\frac{a}{x}\right) \right)^{1+p}}{c a}$$

Result(type 8, 16 leaves):

$$\int x^2 \left( c \text{LambertW}\left(\frac{a}{x}\right) \right)^p dx$$

Problem 77: Unable to integrate problem.

$$\int \frac{\left( c \text{LambertW}\left(\frac{a}{x}\right) \right)^p}{x} dx$$

Optimal(type 4, 38 leaves, 2 steps):

$$\frac{\left(c \operatorname{LambertW}\left(\frac{a}{x}\right)\right)^p}{p} - \frac{\left(c \operatorname{LambertW}\left(\frac{a}{x}\right)\right)^{1+p}}{c(1+p)}$$

Result(type 8, 16 leaves):

$$\int \frac{\left(c \operatorname{LambertW}\left(\frac{a}{x}\right)\right)^p}{x} dx$$

Problem 81: Unable to integrate problem.

$$\int \operatorname{LambertW}(ax^n)^{\frac{-1+n}{n}} dx$$

Optimal(type 4, 40 leaves, 2 steps):

$$\frac{(1-n)x}{\operatorname{LambertW}(ax^n)^{\frac{1}{n}}} + \frac{x}{\operatorname{LambertW}(ax^n)^{\frac{1-n}{n}}}$$

Result(type 8, 16 leaves):

$$\int \operatorname{LambertW}(ax^n)^{\frac{-1+n}{n}} dx$$

Problem 82: Unable to integrate problem.

$$\int \frac{x^{-1-n}}{\sqrt{c \operatorname{LambertW}(ax^n)}} dx$$

Optimal(type 4, 71 leaves, 3 steps):

$$\frac{2a \operatorname{erf}\left(\frac{\sqrt{c \operatorname{LambertW}(ax^n)}}{\sqrt{c}}\right) \sqrt{\pi}}{3n\sqrt{c}} - \frac{2}{3nx^n \sqrt{c \operatorname{LambertW}(ax^n)}} - \frac{2\sqrt{c \operatorname{LambertW}(ax^n)}}{3cnx^n}$$

Result(type 8, 20 leaves):

$$\int \frac{x^{-1-n}}{\sqrt{c \operatorname{LambertW}(ax^n)}} dx$$

Problem 83: Unable to integrate problem.

$$\int x^{-1-2n} (c \operatorname{LambertW}(ax^n))^{11/2} dx$$

Optimal(type 4, 131 leaves, 5 steps):

$$-\frac{165 c^3 (c \operatorname{LambertW}(a x^n))^5 / 2}{128 n x^{2n}} - \frac{55 c^2 (c \operatorname{LambertW}(a x^n))^7 / 2}{32 n x^{2n}} - \frac{11 c (c \operatorname{LambertW}(a x^n))^9 / 2}{8 n x^{2n}} - \frac{(c \operatorname{LambertW}(a x^n))^{11} / 2}{2 n x^{2n}} \\ + \frac{165 a^2 c^{11} / 2 \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{c \operatorname{LambertW}(a x^n)}}{\sqrt{c}}\right) \sqrt{2} \sqrt{\pi}}{512 n}$$

Result(type 8, 20 leaves):

$$\int x^{-1-2n} (c \operatorname{LambertW}(a x^n))^{11} / 2 \, dx$$

Problem 84: Unable to integrate problem.

$$\int x^{-1-2n} (c \operatorname{LambertW}(a x^n))^3 / 2 \, dx$$

Optimal(type 4, 58 leaves, 2 steps):

$$-\frac{2 (c \operatorname{LambertW}(a x^n))^3 / 2}{n x^{2n}} - \frac{3 a^2 c^3 / 2 \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{c \operatorname{LambertW}(a x^n)}}{\sqrt{c}}\right) \sqrt{2} \sqrt{\pi}}{2 n}$$

Result(type 8, 20 leaves):

$$\int x^{-1-2n} (c \operatorname{LambertW}(a x^n))^3 / 2 \, dx$$

Problem 85: Unable to integrate problem.

$$\int \frac{x^{-1+n}}{(c \operatorname{LambertW}(a x^n))^9 / 2} \, dx$$

Optimal(type 4, 111 leaves, 5 steps):

$$-\frac{2 x^n}{7 n (c \operatorname{LambertW}(a x^n))^9 / 2} - \frac{18 x^n}{35 c n (c \operatorname{LambertW}(a x^n))^7 / 2} - \frac{12 x^n}{35 c^2 n (c \operatorname{LambertW}(a x^n))^5 / 2} - \frac{24 x^n}{35 c^3 n (c \operatorname{LambertW}(a x^n))^3 / 2} \\ + \frac{24 \operatorname{erfi}\left(\frac{\sqrt{c \operatorname{LambertW}(a x^n)}}{\sqrt{c}}\right) \sqrt{\pi}}{35 a c^9 / 2 n}$$

Result(type 8, 18 leaves):

$$\int \frac{x^{-1+n}}{(c \operatorname{LambertW}(a x^n))^9 / 2} \, dx$$

Problem 86: Unable to integrate problem.



$$\int \frac{x^{-1+2n}}{\sqrt{c \operatorname{LambertW}(ax^n)}} dx$$

Optimal(type 4, 77 leaves, 3 steps):

$$\frac{cx^{2n}}{8n(c \operatorname{LambertW}(ax^n))^{3/2}} - \frac{\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{c \operatorname{LambertW}(ax^n)}}{\sqrt{c}}\right)\sqrt{2}\sqrt{\pi}}{32a^2n\sqrt{c}} + \frac{x^{2n}}{2n\sqrt{c \operatorname{LambertW}(ax^n)}}$$

Result(type 8, 20 leaves):

$$\int \frac{x^{-1+2n}}{\sqrt{c \operatorname{LambertW}(ax^n)}} dx$$

Problem 87: Unable to integrate problem.

$$\int \frac{x^{-1+2n}}{(c \operatorname{LambertW}(ax^n))^{5/2}} dx$$

Optimal(type 4, 56 leaves, 2 steps):

$$-\frac{2x^{2n}}{n(c \operatorname{LambertW}(ax^n))^{5/2}} + \frac{5 \operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{c \operatorname{LambertW}(ax^n)}}{\sqrt{c}}\right)\sqrt{2}\sqrt{\pi}}{2a^2c^{5/2}n}$$

Result(type 8, 20 leaves):

$$\int \frac{x^{-1+2n}}{(c \operatorname{LambertW}(ax^n))^{5/2}} dx$$

Problem 88: Unable to integrate problem.

$$\int \frac{x^{-1+2n}}{(c \operatorname{LambertW}(ax^n))^{7/2}} dx$$

Optimal(type 4, 79 leaves, 3 steps):

$$-\frac{2x^{2n}}{3n(c \operatorname{LambertW}(ax^n))^{7/2}} - \frac{14x^{2n}}{3cn(c \operatorname{LambertW}(ax^n))^{5/2}} + \frac{14 \operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{c \operatorname{LambertW}(ax^n)}}{\sqrt{c}}\right)\sqrt{2}\sqrt{\pi}}{3a^2c^{7/2}n}$$

Result(type 8, 20 leaves):

$$\int \frac{x^{-1+2n}}{(c \operatorname{LambertW}(ax^n))^{7/2}} dx$$

Problem 89: Unable to integrate problem.

$$\int \frac{x^{-1+2n}}{(c \operatorname{LambertW}(ax^n))^{11/2}} dx$$

Optimal(type 4, 125 leaves, 5 steps):

$$\begin{aligned} & -\frac{2x^{2n}}{7n(c \operatorname{LambertW}(ax^n))^{11/2}} - \frac{22x^{2n}}{35cn(c \operatorname{LambertW}(ax^n))^{9/2}} - \frac{88x^{2n}}{105c^2n(c \operatorname{LambertW}(ax^n))^{7/2}} - \frac{352x^{2n}}{105c^3n(c \operatorname{LambertW}(ax^n))^{5/2}} \\ & + \frac{352 \operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{c \operatorname{LambertW}(ax^n)}}{\sqrt{c}}\right)\sqrt{2}\sqrt{\pi}}{105a^2c^{11/2}n} \end{aligned}$$

Result(type 8, 20 leaves):

$$\int \frac{x^{-1+2n}}{(c \operatorname{LambertW}(ax^n))^{11/2}} dx$$

Problem 90: Unable to integrate problem.

$$\int x^{-1-2n} \operatorname{LambertW}(ax^n)^3 dx$$

Optimal(type 4, 41 leaves, 2 steps):

$$-\frac{3 \operatorname{LambertW}(ax^n)^2}{4nx^{2n}} - \frac{\operatorname{LambertW}(ax^n)^3}{2nx^{2n}}$$

Result(type 8, 18 leaves):

$$\int x^{-1-2n} \operatorname{LambertW}(ax^n)^3 dx$$

Problem 91: Unable to integrate problem.

$$\int \frac{x^{-1+2n}}{\operatorname{LambertW}(ax^n)} dx$$

Optimal(type 4, 37 leaves, 2 steps):

$$\frac{x^{2n}}{4n \operatorname{LambertW}(ax^n)^2} + \frac{x^{2n}}{2n \operatorname{LambertW}(ax^n)}$$

Result(type 8, 18 leaves):

$$\int \frac{x^{-1+2n}}{\operatorname{LambertW}(ax^n)} dx$$

Problem 93: Unable to integrate problem.

$$\int x^{-1+n(2-p)} (c \operatorname{LambertW}(a x^n))^p dx$$

Optimal(type 4, 102 leaves, 3 steps):

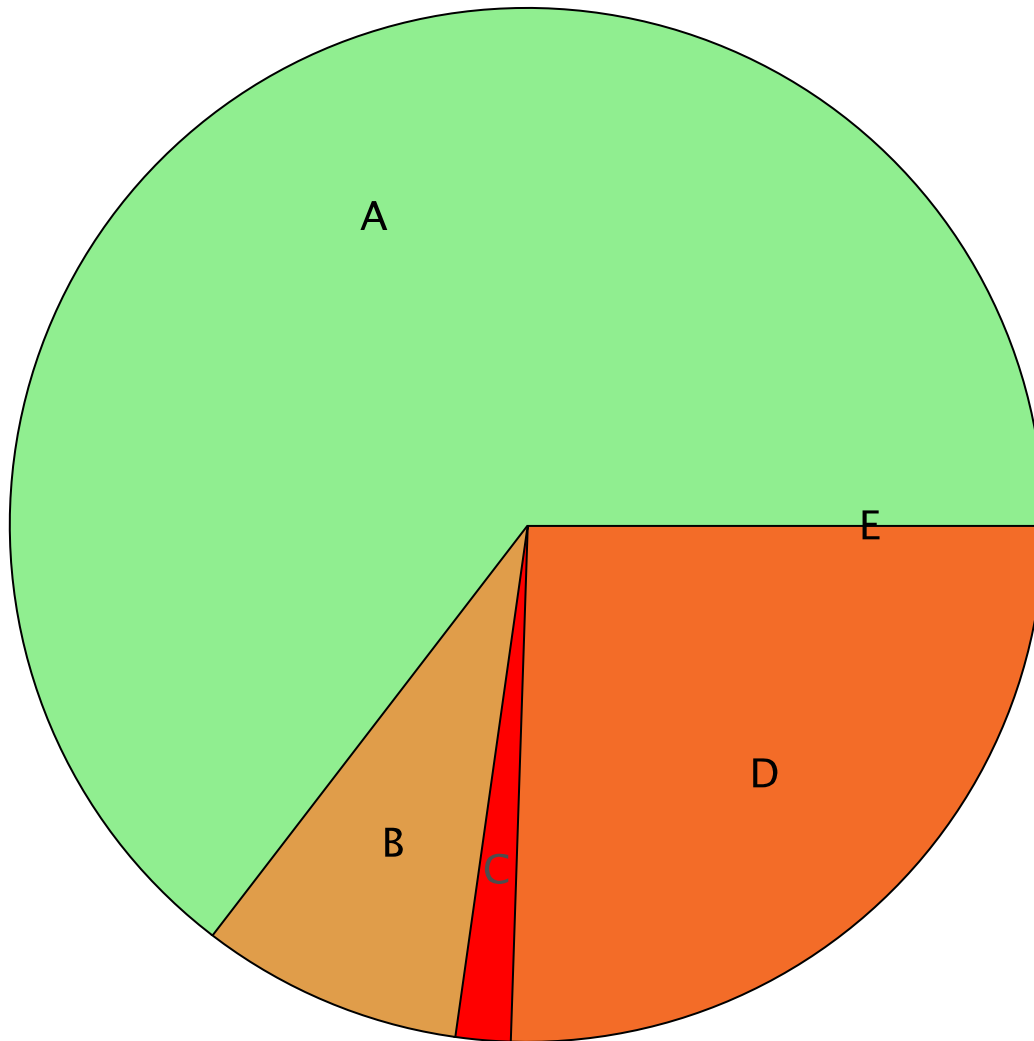
$$\frac{c^2 p x^{n(2-p)} (c \operatorname{LambertW}(a x^n))^{-2+p}}{n(2-p)^3} - \frac{c p x^{n(2-p)} (c \operatorname{LambertW}(a x^n))^{p-1}}{n(2-p)^2} + \frac{x^{n(2-p)} (c \operatorname{LambertW}(a x^n))^p}{n(2-p)}$$

Result(type 8, 24 leaves):

$$\int x^{-1+n(2-p)} (c \operatorname{LambertW}(a x^n))^p dx$$

Summary of Integration Test Results

525 integration problems



A - 339 optimal antiderivatives  
B - 43 more than twice size of optimal antiderivatives  
C - 9 unnecessarily complex antiderivatives  
D - 134 unable to integrate problems  
E - 0 integration timeouts